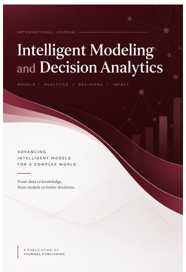

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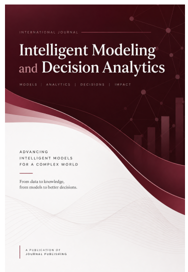

Generalized Interval-Valued p,q-Rung Orthopair Fuzzy Decision-Making Model Integrating Correlation Analysis and Optimization Techniques

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ABSTRACT

Interval-valued p, q-rung orthopair fuzzy (IVpq-ROF) sets extend single-rung orthopair fuzzy models by allowing membership and non-membership grades to be governed by independent rung parameters, thereby widening the space of admissible expert judgements under uncertainty. This paper develops a statistical and decision-analytic framework for this dual-rung setting. Mean, variance, covariance, and correlation coefficients are formulated for IVpq-ROF numbers, and their fundamental properties—including symmetry, boundedness, and behaviour under scalar transformation—are established. Decision-maker weights are derived objectively from the correlation between each expert's assessments and group-level outlier profiles, avoiding subjective prior weighting. A data envelopment analysis (DEA)-inspired linear programming model is then proposed to rank alternatives by optimising each alternative's aggregated membership and non-membership degrees under a shared acceptability threshold, rather than using a fixed set of attribute weights. The resulting ranking is cross-validated through an integrated criteria importance through intercriteria correlation (CRITIC) and exponential Tomada de Decisão Interativa e Multicritério (TODIM) procedure, as well as through a Copeland-based fusion of individual decision-makers' rankings. A sensitivity analysis examines how the ranking responds to independent variation in the two rung parameters. The proposed framework is illustrated through a supplier-type selection problem, and its structure is intended to be broadly applicable to multi-attribute group decision-making problems characterised by uncertain, interval-valued expert judgements.

1. Introduction

Access to timely, adequately resourced healthcare remains uneven across rural and remote communities, where distance from specialist facilities, thin clinical staffing, and unreliable network infrastructure routinely keep patients from the care they need [1]. Telemedicine has long been documented as a means of narrowing this gap, and its more recent incarnation, cloud- and app-based telehealth platforms, has moved the practice from a specialist tool into mainstream clinical use [2,3]. Systematic reviews spanning chronic-disease management, primary care, and mental health report consistent gains in access and patient satisfaction wherever such platforms are adopted [4,5], a pattern the World Health Organization's global eHealth survey identifies as especially pronounced in under-served and

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low-resource settings [6]. Choosing among the many platforms now on offer is nevertheless rarely straightforward: a platform judged strong on clinical grounds may falter where bandwidth is scarce, while one that performs reliably on infrastructure may still strain a resource-constrained health authority's budget [7]. A decision of this kind must jointly weigh clinical, technical, and economic considerations, drawing on the judgement of several experts whose assessments are seldom expressible as precise numbers. Such uncertain, multi-attribute, multi-expert decision-making environments, as exemplified by selecting a telehealth platform for rural healthcare delivery, underscore the need for a robust decision-making framework.

Multi-attribute decision-making (MADM), together with its group-based extension, multi-attribute group decision-making (MAGDM), formalises exactly this class of problem: alternatives are evaluated against several, often conflicting, attributes, and a ranking or selection is sought from the resulting evidence [8]. Because expert judgement under uncertainty rarely reduces to a single crisp number, fuzzy set theory [9], and its successive generalisations have become the natural vehicle for MADM and MAGDM. Atanassov's intuitionistic fuzzy sets [10] paired a membership grade with an independent non-membership grade under an additive bound; Yager's Pythagorean fuzzy sets [11] relaxed this bound to a sum of squares, and Fermatean fuzzy sets [12] relaxed it further to a sum of cubes. Yager's q-rung orthopair fuzzy sets [13] unified this progression by replacing these fixed exponents with a single adjustable rung parameter $q \geq 1$, subsuming the earlier models as special cases and giving experts a correspondingly wider region in which to place their assessments. Recent studies have applied q-ROFSs to information measures for criminal investigation [14], three-way decisions for air-quality evaluation [15], and sustainable urban development [16], while related orthopair extensions such as circular fractional orthopair fuzzy sets continue to broaden this family [17].

A parallel line of development addressed situations where even a single crisp membership value cannot be pinned down, favouring an interval instead. Gorzałczany's interval-valued fuzzy sets [18] and their intuitionistic and Pythagorean counterparts [19, 20] were eventually carried over to the orthopair setting by Joshi *et al.* [21], who introduced interval-valued q-rung orthopair fuzzy sets (IVq-ROFSs). Subsequent work has enriched this model with two-tuple linguistic aggregation operators [22], continuous fusion strategies validated on a smartwatch design case study [23], and weighted averaging operators for MADM [24], confirming IVq-ROFSs as a mature and versatile modelling tool, with further applications reported in metaverse-based digital transformation [25], blockchain-enabled hospital assessment [26], IoT-based sustainable waste management [27], and correlation-driven cloud-service-provider selection [28].

However, one structural limitation persists in the IVq-ROF model: membership and non-membership intervals share a common exponent q, even though expert reasoning does not require the two grades to be scrutinised with the same strictness. Interval-valued p, q-rung orthopair fuzzy sets (IVpq-ROFSs) [29] address this by letting membership and non-membership intervals carry their own rung parameters p and q, widening the admissible region further still. Ali and Khan's foundational work [29] paired this construction with a CRITIC-weighted exponential TODIM ranking procedure and demonstrated it on a green-supplier-selection problem, while Ali's more recent study [30] extended the IVpq-ROF setting with generalized Dombi aggregation operators and applied the resulting model to cybersecurity strategy selection, underscoring the continuing development of this framework and giving this study its point of departure.

A separate strand of research has approached these fuzzy structures from a more statistical angle, treating membership and non-membership grades as quantities whose central tendency, spread, and mutual association can themselves be measured rather than merely aggregated. Correlation coefficients in particular have drawn sustained attention as a way of capturing how two fuzzy assessments move together: Garg [31] derived a correlation coefficient for Pythagorean fuzzy sets and used it directly within a decision-making procedure, and Ning *et al.* [32] extended correlation measures to probabilistic dual hesitant fuzzy sets for MAGDM. Zulqarnain *et al.* [28] went further, showing that such a correlation coefficient can substitute for the usual distance-based closeness computation inside TOPSIS itself, an idea demonstrated on cloud-service-provider selection under interval-valued q-ROF soft sets, while the earlier work of Bonizzoni *et al.* [33] on correlation clustering and consensus clustering illustrates the broader use of pairwise correlation information in data analysis. Singh and Ganie [34] provide the most direct precursor to the present study, defining mean, variance, and correlation coefficients for q-ROFSs from first statistical principles, a foundation this paper carries over to, and extends within, the interval-valued, dual-rung IVpq-ROF setting.

Linear and nonlinear programming have likewise been used to convert fuzzy assessments into a ranking without pre-committing to a fixed attribute-weighting scheme. Chen and Hsu [35] combined a novel score function with a nonlinear programming model for interval-valued intuitionistic fuzzy MADM, and Mehlawat *et al.* [36] developed a nonlinear programming formulation for MADM problems expressed through triangular fuzzy preference and non-preference information. Within intuitionistic and interval-valued intuitionistic fuzzy settings specifically, Riaz *et al.* [37] derived attribute weights through a fairly-aggregation operator coupled with linear programming, Hajiagha *et al.* [38] built an LP model for MAGDM under interval-valued intuitionistic fuzzy information. Yu and Li [39] proposed a goal-programming route for heterogeneous MADM. Closer to the q-rung orthopair setting itself, Yang and Pang [40] combined linear programming with TOPSIS to rank alternatives expressed as hesitant q-rung orthopair fuzzy numbers. Despite this progress, none of these LP-based formulations has yet been extended to a setting where membership and non-membership grades correspond to two independent rung parameters, a gap that motivates the framework developed in this paper.

The principal contributions of this study are as follows:

1. Mean, variance, covariance, and correlation coefficients are formulated for interval-valued p, q-rung orthopair fuzzy numbers, extending existing statistical treatments of q-ROFSs to a setting where membership and non-membership grades carry independent rung parameters.
2. Decision-maker weights are derived objectively from each expert's correlation with group-level upper and lower outlier profiles, removing the need to fix these weights subjectively in advance.
3. A DEA-inspired linear programming model is formulated that ranks alternatives by optimizing their own membership and non-membership degrees under a shared acceptability threshold, without committing to a fixed attribute-weighting scheme.
4. The resulting ranking is cross-validated through an integrated CRITIC and Exponential TODIM procedure together with a Copeland fusion of individual decision-makers' rankings, and its sensitivity to the two rung parameters is examined independently.
5. The complete framework is applied to a rural telehealth-platform selection problem, illustrating how the proposed statistical and optimization machinery translates into a defensible ranking under multi-expert, interval-valued uncertainty.

The remainder of this paper is organized as follows. Section 2 presents the basic definitions and operations of IVpq-ROFSs. Section 3 extends the notions of mean, variance, covariance, and correlation to this setting. Section 4 details the determination of experts' weights and the formulation of the LP-based ranking model. Section 5 applies the proposed framework to a telehealth-platform selection problem and validates the resulting ranking through CRITIC–Exp-TODIM and Copeland rank fusion. Section 6 examines the sensitivity of the ranking to the rung parameters p and q, and Section 7 concludes the paper.

2. Basic Concepts

Before developing the correlation-based weighting scheme and the linear-programming ranking procedure, it is necessary to first establish the underlying fuzzy structure, its arithmetic operations, the two scalar comparison functions, and the aggregation rule on which the proposed framework rests.

Definition 1. *[29] Let X be a nonempty universal set. An* interval-valued p, q-rung orthopair fuzzy set *(IVpq-ROFS) Γ constructed over X is given by*

$$\Gamma = \{\langle x,\ [\mathsf{a}(x), \mathsf{b}(x)],\ [\mathsf{c}(x), \mathsf{d}(x)]\rangle \mid x \in X\}, \qquad p, q \geq 1, \tag{1}$$

where, for every $x \in X$, the interval $[\mathsf{a}(x), \mathsf{b}(x)] \subseteq [0,1]$ records the membership grade assigned to x and the interval $[\mathsf{c}(x), \mathsf{d}(x)] \subseteq [0,1]$ records the non-membership grade, the two being linked through the requirement $0 \leq \big(\mathsf{b}(x)\big)^p + \big(\mathsf{d}(x)\big)^q \leq 1$. Allowing the membership component to carry its own exponent p and the non-membership component a possibly different exponent q is precisely what distinguishes this structure from its single-exponent predecessor and grants it a wider admissible region in which experts may position their judgements.

For a single fixed element the set reduces to the pair $\Gamma = \big\langle [\mathsf{a}, \mathsf{b}], [\mathsf{c}, \mathsf{d}] \big\rangle$, referred to as an IV$pq$-ROFN, and it must satisfy $\big(\mathsf{b}\big)^p + \big(\mathsf{d}\big)^q \leq 1$, together with $\mathsf{a} \leq \mathsf{b}$ and $\mathsf{c} \leq \mathsf{d}$.

The residual uncertainty that neither the membership nor the non-membership interval accounts for is captured by the hesitancy degree

$$\pi = \left[\sqrt[\ell]{1 - (\mathsf{a})^p - (\mathsf{c})^q}\ ,\ \sqrt[\ell]{1 - (\mathsf{b})^p - (\mathsf{d})^q} \right], \tag{2}$$

where $\ell = \mathrm{lcm}(p, q)$ is taken so that a single common root can be applied consistently on both sides of the interval even when p and q differ.

Definition 2. *[29] Let $\Gamma_1 = \big\langle [\mathsf{a}_1, \mathsf{b}_1], [\mathsf{c}_1, \mathsf{d}_1] \big\rangle$ and $\Gamma_2 = \big\langle [\mathsf{a}_2, \mathsf{b}_2], [\mathsf{c}_2, \mathsf{d}_2] \big\rangle$ be two IVpq-ROFNs, and let $\Gamma = \big\langle [\mathsf{a}, \mathsf{b}], [\mathsf{c}, \mathsf{d}] \big\rangle$ be a third such number, with $\lambda > 0$ an arbitrary positive scalar. The arithmetic on these numbers is fixed by the following rules.*

1. $\Gamma_1 \oplus \Gamma_2 = \left\langle \left[\left(\mathsf{a}_1^p + \mathsf{a}_2^p - \mathsf{a}_1^p \mathsf{a}_2^p \right)^{1/p}, \left(\mathsf{b}_1^p + \mathsf{b}_2^p - \mathsf{b}_1^p \mathsf{b}_2^p \right)^{1/p} \right],\ [\mathsf{c}_1 \mathsf{c}_2,\ \mathsf{d}_1 \mathsf{d}_2] \right\rangle;$
2. $\Gamma_1 \otimes \Gamma_2 = \left\langle [\mathsf{a}_1 \mathsf{a}_2,\ \mathsf{b}_1 \mathsf{b}_2],\ \left[\left(\mathsf{c}_1^q + \mathsf{c}_2^q - \mathsf{c}_1^q \mathsf{c}_2^q \right)^{1/q}, \left(\mathsf{d}_1^q + \mathsf{d}_2^q - \mathsf{d}_1^q \mathsf{d}_2^q \right)^{1/q} \right] \right\rangle;$
3. $\Gamma^\lambda = \left\langle \left[(\mathsf{a})^\lambda, (\mathsf{b})^\lambda \right],\ \left[\left(1 - \left(1 - \mathsf{c}^q \right)^\lambda \right)^{1/q}, \left(1 - \left(1 - \mathsf{d}^q \right)^\lambda \right)^{1/q} \right] \right\rangle;$
4. $\lambda\Gamma = \left\langle \left[\left(1 - \left(1 - \mathsf{a}^p \right)^\lambda \right)^{1/p}, \left(1 - \left(1 - \mathsf{b}^p \right)^\lambda \right)^{1/p} \right],\ \left[(\mathsf{c})^\lambda, (\mathsf{d})^\lambda \right] \right\rangle;$
5. $\Gamma^c = \langle [\mathsf{c}, \mathsf{d}],\ [\mathsf{a}, \mathsf{b}] \rangle.$

Definition 3. *[29] For an IVpq-ROFN $\Gamma = \big\langle [\mathsf{a}, \mathsf{b}], [\mathsf{c}, \mathsf{d}] \big\rangle$, the score function, which condenses the interval information into a single comparable number, is defined as*

$$S(\Gamma) = \frac{1}{3} \left(1 + \frac{\mathsf{a}^p + \mathsf{b}^p - \mathsf{c}^q - \mathsf{d}^q}{2} + \frac{\mathsf{a}^p \big(1 - \mathsf{c}^q\big) + \mathsf{b}^p \big(1 - \mathsf{d}^q\big)}{2 + \mathsf{c}^q + \mathsf{d}^q} \right), \tag{3}$$

where $p, q \in [1, \infty)$ and $S(\Gamma) \in [0,1]$. Between any two IVpq-ROFNs, the one carrying the larger score value is regarded as the more favourable.

Definition 4. *[29] The accuracy degree of an IVpq-ROFN Γ is given by*

$$A(\Gamma) = \frac{1}{2} \left(\mathsf{a}^p + \mathsf{b}^p + \mathsf{c}^q + \mathsf{d}^q \right), \qquad A(\Gamma) \in [0,1]. \tag{4}$$

Whenever two IVpq-ROFNs happen to share an identical score, the one possessing the higher accuracy degree is taken to be the greater of the two, thereby resolving the tie left by the score function alone.

Definition 5. *[29] Let $\Gamma_i = \big\langle [\mathsf{a}_i, \mathsf{b}_i], [\mathsf{c}_i, \mathsf{d}_i] \big\rangle$, $i = 1, 2, \ldots, n$, be a collection of IVpq-ROFNs, and let $w = (w_1, w_2, \ldots, w_n)^T$ be an associated weighting vector satisfying $w_i \in [0,1]$ and $\sum_{i=1}^n w_i = 1$. The interval-valued q-rung orthopair fuzzy weighted average (IVpq-ROFWA) operator, which fuses these n*

numbers into a single representative one, is given by

$$\text{IV}pq\text{-ROFWA}(\Gamma_1, \Gamma_2, \ldots, \Gamma_n) = \bigoplus_{i=1}^{n} w_i \Gamma_i \tag{5}$$

which, upon expanding the summation through the addition rule of Definition 2, works out to

$$\text{IV}pq\text{-ROFWA}(\Gamma_1, \Gamma_2, \ldots, \Gamma_n) = \left\langle \begin{array}{c} \left[\sqrt[p]{1 - \prod_{i=1}^{n} \left(1 - (\mathsf{a}_i)^p\right)^{w_i}},\ \sqrt[p]{1 - \prod_{i=1}^{n} \left(1 - (\mathsf{b}_i)^p\right)^{w_i}} \right], \\ \left[\prod_{i=1}^{n} \left(\mathsf{c}_i\right)^{w_i},\ \prod_{i=1}^{n} \left(\mathsf{d}_i\right)^{w_i} \right] \end{array} \right\rangle. \tag{6}$$

3. Development of Statistical Measures

In this section, we extend the notions of variance, covariance, and correlation coefficient to IVpq-ROFSs, adapting these classical statistical measures to the interval-valued, dual-rung structure of the membership and non-membership grades.

Definition 6. *Let Γ be an IVpq-ROFS containing m elements $x_1, x_2, \ldots, x_m$, with membership interval $[\mathsf{a}_\Gamma(x_k), \mathsf{b}_\Gamma(x_k)]$ and non-membership interval $[\mathsf{c}_\Gamma(x_k), \mathsf{d}_\Gamma(x_k)]$ attached to each element x_k. The average of Γ is obtained by averaging each of the four bounds separately over all m elements:*

$$E(\Gamma) = \left\langle [\bar{\mathsf{a}}_\Gamma, \bar{\mathsf{b}}_\Gamma], [\bar{\mathsf{c}}_\Gamma, \bar{\mathsf{d}}_\Gamma] \right\rangle = \left\langle \left[\frac{1}{m} \sum_{k=1}^{m} \mathsf{a}_\Gamma(x_k),\ \frac{1}{m} \sum_{k=1}^{m} \mathsf{b}_\Gamma(x_k) \right], \left[\frac{1}{m} \sum_{k=1}^{m} \mathsf{c}_\Gamma(x_k),\ \frac{1}{m} \sum_{k=1}^{m} \mathsf{d}_\Gamma(x_k) \right] \right\rangle. \tag{7}$$

Definition 7. *For the same IVpq-ROFS Γ with m elements, the variance of Γ, for the rung parameters $p, q \geq 1$, is defined as follows:*

$$V(\Gamma) = \frac{1}{2(m-1)} \sum_{k=1}^{m} \left\{ \begin{array}{c} \left(\mathsf{a}_\Gamma(x_k)^p - \bar{\mathsf{a}}_\Gamma^p\right)^2 + \left(\mathsf{b}_\Gamma(x_k)^p - \bar{\mathsf{b}}_\Gamma^p\right)^2 \\ + \left(\mathsf{c}_\Gamma(x_k)^q - \bar{\mathsf{c}}_\Gamma^q\right)^2 + \left(\mathsf{d}_\Gamma(x_k)^q - \bar{\mathsf{d}}_\Gamma^q\right)^2 \end{array} \right\}. \tag{8}$$

The membership bounds are thus compared through the exponent p, while the non-membership bounds are compared through the exponent q, consistent with the rung convention adopted throughout this manuscript.

Definition 8. *Let Γ and Λ be two IVpq-ROFSs defined over the same m elements. The covariance between Γ and Λ, for $p, q \geq 1$, is given by*

$$C(\Gamma, \Lambda) = \frac{1}{2(m-1)} \sum_{k=1}^{m} \left\{ \begin{array}{c} \left(\mathsf{a}_\Gamma(x_k)^p - \bar{\mathsf{a}}_\Gamma^p\right)\left(\mathsf{a}_\Lambda(x_k)^p - \bar{\mathsf{a}}_\Lambda^p\right) \\ + \left(\mathsf{b}_\Gamma(x_k)^p - \bar{\mathsf{b}}_\Gamma^p\right)\left(\mathsf{b}_\Lambda(x_k)^p - \bar{\mathsf{b}}_\Lambda^p\right) \\ + \left(\mathsf{c}_\Gamma(x_k)^q - \bar{\mathsf{c}}_\Gamma^q\right)\left(\mathsf{c}_\Lambda(x_k)^q - \bar{\mathsf{c}}_\Lambda^q\right) \\ + \left(\mathsf{d}_\Gamma(x_k)^q - \bar{\mathsf{d}}_\Gamma^q\right)\left(\mathsf{d}_\Lambda(x_k)^q - \bar{\mathsf{d}}_\Lambda^q\right) \end{array} \right\}. \tag{9}$$

Proposition 1. *For any two IVpq-ROFSs Γ and Λ containing m elements each, the following hold:*

1. $C(\Gamma, \Lambda) = C(\Lambda, \Gamma)$;
2. $C(\Gamma, \Gamma) = V(\Gamma)$;
3. $|C(\Gamma, \Lambda)| \leq \sqrt{V(\Gamma)\, V(\Lambda)}$.

Proof. **(1)** We have

$$C(\Gamma,\Lambda) = \frac{1}{2(m-1)}\sum_{k=1}^{m}\left\{\begin{array}{c}\left(\mathsf{a}_\Gamma(x_k)^p-\bar{\mathsf{a}}_\Gamma^p\right)\left(\mathsf{a}_\Lambda(x_k)^p-\bar{\mathsf{a}}_\Lambda^p\right)\\+\left(\mathsf{b}_\Gamma(x_k)^p-\bar{\mathsf{b}}_\Gamma^p\right)\left(\mathsf{b}_\Lambda(x_k)^p-\bar{\mathsf{b}}_\Lambda^p\right)\\+\left(\mathsf{c}_\Gamma(x_k)^q-\bar{\mathsf{c}}_\Gamma^q\right)\left(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\right)\\+\left(\mathsf{d}_\Gamma(x_k)^q-\bar{\mathsf{d}}_\Gamma^q\right)\left(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\right)\end{array}\right\},$$

$$= \frac{1}{2(m-1)}\sum_{k=1}^{m}\left\{\begin{array}{c}\left(\mathsf{a}_\Lambda(x_k)^p-\bar{\mathsf{a}}_\Lambda^p\right)\left(\mathsf{a}_\Gamma(x_k)^p-\bar{\mathsf{a}}_\Gamma^p\right)\\+\left(\mathsf{b}_\Lambda(x_k)^p-\bar{\mathsf{b}}_\Lambda^p\right)\left(\mathsf{b}_\Gamma(x_k)^p-\bar{\mathsf{b}}_\Gamma^p\right)\\+\left(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\right)\left(\mathsf{c}_\Gamma(x_k)^q-\bar{\mathsf{c}}_\Gamma^q\right)\\+\left(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\right)\left(\mathsf{d}_\Gamma(x_k)^q-\bar{\mathsf{d}}_\Gamma^q\right)\end{array}\right\},$$

$$= C(\Lambda,\Gamma).$$

(2) From Eq. (9),

$$C(\Gamma,\Gamma) = \frac{1}{2(m-1)}\sum_{k=1}^{m}\left\{\begin{array}{c}\left(\mathsf{a}_\Gamma(x_k)^p-\bar{\mathsf{a}}_\Gamma^p\right)\left(\mathsf{a}_\Gamma(x_k)^p-\bar{\mathsf{a}}_\Gamma^p\right)\\+\left(\mathsf{b}_\Gamma(x_k)^p-\bar{\mathsf{b}}_\Gamma^p\right)\left(\mathsf{b}_\Gamma(x_k)^p-\bar{\mathsf{b}}_\Gamma^p\right)\\+\left(\mathsf{c}_\Gamma(x_k)^q-\bar{\mathsf{c}}_\Gamma^q\right)\left(\mathsf{c}_\Gamma(x_k)^q-\bar{\mathsf{c}}_\Gamma^q\right)\\+\left(\mathsf{d}_\Gamma(x_k)^q-\bar{\mathsf{d}}_\Gamma^q\right)\left(\mathsf{d}_\Gamma(x_k)^q-\bar{\mathsf{d}}_\Gamma^q\right)\end{array}\right\}$$

$$= \frac{1}{2(m-1)}\sum_{k=1}^{m}\left\{\begin{array}{c}\left(\mathsf{a}_\Gamma(x_k)^p-\bar{\mathsf{a}}_\Gamma^p\right)^2+\left(\mathsf{b}_\Gamma(x_k)^p-\bar{\mathsf{b}}_\Gamma^p\right)^2\\+\left(\mathsf{c}_\Gamma(x_k)^q-\bar{\mathsf{c}}_\Gamma^q\right)^2+\left(\mathsf{d}_\Gamma(x_k)^q-\bar{\mathsf{d}}_\Gamma^q\right)^2\end{array}\right\}$$

$$= V(\Gamma).$$

(3) Using the Cauchy–Schwarz inequality, we have

$$\left(C(\Gamma,\Lambda)\right)^2 = \frac{1}{2(m-1)}\sum_{k=1}^{m}\left\{\begin{array}{c}\left(\mathsf{a}_\Gamma(x_k)^p-\bar{\mathsf{a}}_\Gamma^p\right)\left(\mathsf{a}_\Lambda(x_k)^p-\bar{\mathsf{a}}_\Lambda^p\right)\\+\left(\mathsf{b}_\Gamma(x_k)^p-\bar{\mathsf{b}}_\Gamma^p\right)\left(\mathsf{b}_\Lambda(x_k)^p-\bar{\mathsf{b}}_\Lambda^p\right)\\+\left(\mathsf{c}_\Gamma(x_k)^q-\bar{\mathsf{c}}_\Gamma^q\right)\left(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\right)\\+\left(\mathsf{d}_\Gamma(x_k)^q-\bar{\mathsf{d}}_\Gamma^q\right)\left(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\right)\end{array}\right\}^2$$

$$\leq \frac{1}{2(m-1)}\sum_{k=1}^{m}\left\{\begin{array}{c}\left(\mathsf{a}_\Gamma(x_k)^p-\bar{\mathsf{a}}_\Gamma^p\right)^2+\left(\mathsf{b}_\Gamma(x_k)^p-\bar{\mathsf{b}}_\Gamma^p\right)^2\\+\left(\mathsf{c}_\Gamma(x_k)^q-\bar{\mathsf{c}}_\Gamma^q\right)^2+\left(\mathsf{d}_\Gamma(x_k)^q-\bar{\mathsf{d}}_\Gamma^q\right)^2\end{array}\right\}$$

$$\times \frac{1}{2(m-1)}\sum_{k=1}^{m}\left\{\begin{array}{c}\left(\mathsf{a}_\Lambda(x_k)^p-\bar{\mathsf{a}}_\Lambda^p\right)^2+\left(\mathsf{b}_\Lambda(x_k)^p-\bar{\mathsf{b}}_\Lambda^p\right)^2\\+\left(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\right)^2+\left(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\right)^2\end{array}\right\}$$

$$= V(\Gamma)\,V(\Lambda).$$

$$|C(\Gamma,\Lambda)| \leq \sqrt{V(\Gamma)\,V(\Lambda)}.$$

□

Next, we turn to the correlation coefficient linking two IVpq-ROFSs.

Definition 9. *For any two IVpq-ROFSs Γ and Λ having m elements, the correlation coefficient between Γ and Λ, for the rung parameters $p, q \geq 1$, is defined as*

$$R(\Gamma,\Lambda) = \frac{C(\Gamma,\Lambda)}{\sqrt{V(\Gamma)\,V(\Lambda)}}$$

$$= \frac{\frac{1}{2(m-1)}\sum_{k=1}^{m}\left(\begin{array}{c}\left(\mathsf{a}_\Gamma(x_k)^p-\bar{\mathsf{a}}_\Gamma^p\right)\left(\mathsf{a}_\Lambda(x_k)^p-\bar{\mathsf{a}}_\Lambda^p\right)+\left(\mathsf{b}_\Gamma(x_k)^p-\bar{\mathsf{b}}_\Gamma^p\right)\left(\mathsf{b}_\Lambda(x_k)^p-\bar{\mathsf{b}}_\Lambda^p\right)\\+\left(\mathsf{c}_\Gamma(x_k)^q-\bar{\mathsf{c}}_\Gamma^q\right)\left(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\right)+\left(\mathsf{d}_\Gamma(x_k)^q-\bar{\mathsf{d}}_\Gamma^q\right)\left(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\right)\end{array}\right)}{\sqrt{\frac{1}{2(m-1)}\sum_{k=1}^{m}\left(\begin{array}{c}\left(\mathsf{a}_\Gamma(x_k)^p-\bar{\mathsf{a}}_\Gamma^p\right)^2\\+\left(\mathsf{b}_\Gamma(x_k)^p-\bar{\mathsf{b}}_\Gamma^p\right)^2\\+\left(\mathsf{c}_\Gamma(x_k)^q-\bar{\mathsf{c}}_\Gamma^q\right)^2\\+\left(\mathsf{d}_\Gamma(x_k)^q-\bar{\mathsf{d}}_\Gamma^q\right)^2\end{array}\right)\times\frac{1}{2(m-1)}\sum_{k=1}^{m}\left(\begin{array}{c}\left(\mathsf{a}_\Lambda(x_k)^p-\bar{\mathsf{a}}_\Lambda^p\right)^2\\+\left(\mathsf{b}_\Lambda(x_k)^p-\bar{\mathsf{b}}_\Lambda^p\right)^2\\+\left(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\right)^2\\+\left(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\right)^2\end{array}\right)}}. \quad (10)$$

The correlation coefficient plays a central role in gauging how strongly two imprecise, fuzzy-valued quantities are related, and it offers a principled tool for reasoning about dependence in uncertain settings where classical, crisp mathematical machinery would otherwise fall short.

Example 1. *Consider two IVpq-ROFSs, each containing two elements, with $p = 2$ and $q = 3$:*

$$\Gamma = \big(\langle[0.55, 0.85], [0.35, 0.60]\rangle,\ \langle[0.65, 0.90], [0.30, 0.55]\rangle\big),$$

$$\Lambda = \big(\langle[0.50, 0.80], [0.40, 0.65]\rangle,\ \langle[0.60, 0.88], [0.32, 0.58]\rangle\big).$$

Mean values. *Averaging the corresponding bounds across the two elements gives*

$$\bar{\Gamma} = \langle[0.60, 0.875], [0.325, 0.575]\rangle, \qquad \bar{\Lambda} = \langle[0.55, 0.84], [0.36, 0.615]\rangle.$$

Variances. *Using Eq.* (8) *with $p = 2$ for the membership bounds and $q = 3$ for the non-membership bounds, the variance of Γ is*

$$V(\Gamma) = \frac{1}{2(2-1)}\left[\begin{array}{c}(0.55^2-0.60^2)^2+(0.85^2-0.875^2)^2+(0.35^3-0.325^3)^2+(0.60^3-0.575^3)^2\\+(0.65^2-0.60^2)^2+(0.90^2-0.875^2)^2+(0.30^3-0.325^3)^2+(0.55^3-0.575^3)^2\end{array}\right]$$
$$= 0.0062.$$

Similarly, the variance of Λ is

$$V(\Lambda) = \frac{1}{2(2-1)}\left[\begin{array}{c}(0.50^2-0.55^2)^2+(0.80^2-0.84^2)^2+(0.40^3-0.36^3)^2+(0.65^3-0.615^3)^2\\+(0.60^2-0.55^2)^2+(0.88^2-0.84^2)^2+(0.32^3-0.36^3)^2+(0.58^3-0.615^3)^2\end{array}\right]$$
$$= 0.0094.$$

Covariance. *Using Eq.* (9)*, the covariance is*

$$C(\Gamma,\Lambda) = \frac{1}{2(2-1)}\left[\begin{array}{c}(0.55^2-0.60^2)(0.50^2-0.55^2)+(0.85^2-0.875^2)(0.80^2-0.84^2)\\+(0.35^3-0.325^3)(0.40^3-0.36^3)+(0.60^3-0.575^3)(0.65^3-0.615^3)\\+(0.65^2-0.60^2)(0.60^2-0.55^2)+(0.90^2-0.875^2)(0.88^2-0.84^2)\\+(0.30^3-0.325^3)(0.32^3-0.36^3)+(0.55^3-0.575^3)(0.58^3-0.615^3)\end{array}\right]$$
$$= 0.0074.$$

Correlation coefficient. *Using Eq.* (10)*, the correlation between Γ and Λ is*

$$R(\Gamma,\Lambda) = \frac{C(\Gamma,\Lambda)}{\sqrt{V(\Gamma)\,V(\Lambda)}} = \frac{0.0074}{\sqrt{0.0062\times 0.0094}} = 0.9693.$$

This near-unit correlation reflects a close linear relationship between the rung-transformed membership and non-membership values of the two IVpq-ROFSs Γ and Λ.

Theorem 1. *For any two IVpq-ROFSs Γ and Λ, the following hold:*

1. $R(\Gamma,\Lambda)=R(\Lambda,\Gamma)$;
2. $-1\le R(\Gamma,\Lambda)\le 1$;
3. If $\Gamma=\$\Lambda$ for some scalar $\$$, with the rung parameters coinciding, i.e. $p=q$, then

$$R(\Gamma,\Lambda)=\begin{cases}1, & \text{if } \$>0,\\ -1, & \text{if } \$<0 \text{ and } q=2n+1,\ n\in\mathbb{Z}^{+},\\ 1, & \text{if } \$<0 \text{ and } q=2n,\ n\in\mathbb{Z}^{+}.\end{cases}$$

Proof. **(1)** Interchanging the labels Γ and Λ throughout Eq. (10) leaves the expression unaltered, so $R(\Gamma,\Lambda)=R(\Lambda,\Gamma)$ follows at once.

(2) By Proposition 1,

$$|C(\Gamma,\Lambda)|\le\sqrt{V(\Gamma)\,V(\Lambda)}\implies -\sqrt{V(\Gamma)\,V(\Lambda)}\le C(\Gamma,\Lambda)\le\sqrt{V(\Gamma)\,V(\Lambda)}$$
$$\implies -1\le\frac{C(\Gamma,\Lambda)}{\sqrt{V(\Gamma)\,V(\Lambda)}}\le 1\implies -1\le R(\Gamma,\Lambda)\le 1.$$

(3) Let $\Gamma=\$\Lambda$ with $p=q$, i.e. $\mathsf{a}_\Gamma(x_k)=\$\,\mathsf{a}_\Lambda(x_k)$, $\mathsf{b}_\Gamma(x_k)=\$\,\mathsf{b}_\Lambda(x_k)$, $\mathsf{c}_\Gamma(x_k)=\$\,\mathsf{c}_\Lambda(x_k)$, and $\mathsf{d}_\Gamma(x_k)=\$\,\mathsf{d}_\Lambda(x_k)$. Then, from Eq. (10),

$$R(\Gamma,\Lambda)=$$
$$\frac{\frac{1}{2(m-1)}\sum_{k=1}^{m}\left(\begin{array}{c}\Big((\$\mathsf{a}_\Lambda(x_k))^q-\$^q\bar{\mathsf{a}}_\Lambda^q\Big)\Big(\mathsf{a}_\Lambda(x_k)^q-\bar{\mathsf{a}}_\Lambda^q\Big)+\Big((\$\mathsf{b}_\Lambda(x_k))^q-\$^q\bar{\mathsf{b}}_\Lambda^q\Big)\Big(\mathsf{b}_\Lambda(x_k)^q-\bar{\mathsf{b}}_\Lambda^q\Big)\\+\Big((\$\mathsf{c}_\Lambda(x_k))^q-\$^q\bar{\mathsf{c}}_\Lambda^q\Big)\Big(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\Big)+\Big((\$\mathsf{d}_\Lambda(x_k))^q-\$^q\bar{\mathsf{d}}_\Lambda^q\Big)\Big(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\Big)\end{array}\right)}{\sqrt{\frac{1}{2(m-1)}\sum_{k=1}^{m}\left(\begin{array}{c}\Big((\$\mathsf{a}_\Lambda(x_k))^q-\$^q\bar{\mathsf{a}}_\Lambda^q\Big)^2\\+\Big((\$\mathsf{b}_\Lambda(x_k))^q-\$^q\bar{\mathsf{b}}_\Lambda^q\Big)^2\\+\Big((\$\mathsf{c}_\Lambda(x_k))^q-\$^q\bar{\mathsf{c}}_\Lambda^q\Big)^2\\+\Big((\$\mathsf{d}_\Lambda(x_k))^q-\$^q\bar{\mathsf{d}}_\Lambda^q\Big)^2\end{array}\right)\times\frac{1}{2(m-1)}\sum_{k=1}^{m}\left(\begin{array}{c}\Big(\mathsf{a}_\Lambda(x_k)^q-\bar{\mathsf{a}}_\Lambda^q\Big)^2\\+\Big(\mathsf{b}_\Lambda(x_k)^q-\bar{\mathsf{b}}_\Lambda^q\Big)^2\\+\Big(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\Big)^2\\+\Big(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\Big)^2\end{array}\right)}}$$

$$=\frac{\frac{\$^q}{2(m-1)}\sum_{k=1}^{m}\left(\begin{array}{c}\Big(\mathsf{a}_\Lambda(x_k)^q-\bar{\mathsf{a}}_\Lambda^q\Big)^2+\Big(\mathsf{b}_\Lambda(x_k)^q-\bar{\mathsf{b}}_\Lambda^q\Big)^2\\+\Big(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\Big)^2+\Big(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\Big)^2\end{array}\right)}{\frac{\$^q}{2(m-1)}\sum_{k=1}^{m}\left(\begin{array}{c}\Big(\mathsf{a}_\Lambda(x_k)^q-\bar{\mathsf{a}}_\Lambda^q\Big)^2+\Big(\mathsf{b}_\Lambda(x_k)^q-\bar{\mathsf{b}}_\Lambda^q\Big)^2\\+\Big(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\Big)^2+\Big(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\Big)^2\end{array}\right)}=1.$$

Now let $\$<0$, and write $\$=-\$'$ with $\$'>0$. Then

$$R(\Gamma,\Lambda)=\frac{\frac{(-1)^q\$'^q}{2(m-1)}\sum_{k=1}^{m}\left(\begin{array}{c}\Big(\mathsf{a}_\Lambda(x_k)^q-\bar{\mathsf{a}}_\Lambda^q\Big)^2+\Big(\mathsf{b}_\Lambda(x_k)^q-\bar{\mathsf{b}}_\Lambda^q\Big)^2\\+\Big(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\Big)^2+\Big(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\Big)^2\end{array}\right)}{\frac{\$'^q}{2(m-1)}\sum_{k=1}^{m}\left(\begin{array}{c}\Big(\mathsf{a}_\Lambda(x_k)^q-\bar{\mathsf{a}}_\Lambda^q\Big)^2+\Big(\mathsf{b}_\Lambda(x_k)^q-\bar{\mathsf{b}}_\Lambda^q\Big)^2\\+\Big(\mathsf{c}_\Lambda(x_k)^q-\bar{\mathsf{c}}_\Lambda^q\Big)^2+\Big(\mathsf{d}_\Lambda(x_k)^q-\bar{\mathsf{d}}_\Lambda^q\Big)^2\end{array}\right)}=(-1)^q$$

$$=\begin{cases}-1, & \text{if } \$<0 \text{ and } q=2n+1,\ n\in\mathbb{Z}^{+},\\ 1, & \text{if } \$<0 \text{ and } q=2n,\ n\in\mathbb{Z}^{+}.\end{cases}$$

□

Remark 1. *Throughout this study, the mean, variance, covariance, and correlation coefficient attached* *to IVpq-ROFNs are built solely from the membership and non-membership bounds; the hesitancy degree is deliberately left out of these statistical constructions. This is not an omission but a consequence of how π is defined in Eq.* (2)*: since $\pi = \left[\sqrt[\ell]{1-\mathsf{a}^p-\mathsf{c}^q},\ \sqrt[\ell]{1-\mathsf{b}^p-\mathsf{d}^q}\right]$ with $\ell = \mathrm{lcm}(p,q)$, the hesitancy interval is completely determined once the membership and non-membership bounds are fixed, and therefore carries no information beyond what* $\mathsf{a},\mathsf{b},\mathsf{c},\mathsf{d}$ *already encode. Folding it back into the statistical measures would only duplicate existing information while adding unnecessary computational overhead. Omitting it keeps the resulting measures easier to interpret.*

4. Proposed Methodology

This section presents the proposed methodology for tackling the MAGDM problem under the IVpq-ROFNs framework, including the necessary mathematical procedures and decision-making steps.

4.1 Experts' weights

Suppose k decision-makers (DMs) jointly assess a set of alternatives $\aleph = \{S_1, S_2, \ldots, S_n\}$ against a collection of attributes $\zeta = \{\zeta_1, \zeta_2, \ldots, \zeta_m\}$. Each DM $r \in \{1, 2, \ldots, k\}$ supplies an individual decision matrix $D^r = (\Gamma^r_{ij})_{n\times m}$, in which the entry Γ^r_{ij} records how the rth DM rates the ith alternative on the jth attribute. Every such rating is expressed as an IVpq-ROFN,

$$\Gamma^r_{ij} = \left\langle [\mathsf{a}^r_{ij}, \mathsf{b}^r_{ij}], [\mathsf{c}^r_{ij}, \mathsf{d}^r_{ij}] \right\rangle, \qquad i = 1, \ldots, n;\ j = 1, \ldots, m;\ r = 1, \ldots, k. \tag{11}$$

The task is to fuse these k individual opinions into a single consensus ranking of the n alternatives. A MAGDM problem of this kind unfolds in three stages: fixing the relative influence of the DMs, fixing the relative importance of the attributes, and finally producing the ranking itself. A large part of the existing literature treats the first two stages as already settled, typically by fixing the DM and attribute weights in advance; such a choice, however, can quietly inject the analyst's own bias into what is meant to be an objective exercise. A sounder route is to let both sets of weights emerge directly from the assessment data itself.

Classical TOPSIS separates alternatives according to how close, in a distance sense, they lie to a Positive Ideal Solution (PIS) and how far they lie from a Negative Ideal Solution (NIS). Here, we depart from that distance-based logic and instead gauge closeness through correlation: a DM (or, later, an alternative) is preferred when its ratings correlate positively with the most favourable reference profile and negatively with the least favourable one. This correlation-based reading tends to behave more predictably once membership and non-membership values are only known up to an interval.

To fix the DM weights along these lines, we build two reference matrices — an upper outlier assessment (UOA) and a lower outlier assessment (LOA) — by comparing, attribute by attribute and alternative by alternative, how far each DM's rating sits from the rest of the panel:

$$D^+ = \left(\Gamma^+_{ij}\right)_{n\times m}, \qquad D^- = \left(\Gamma^-_{ij}\right)_{n\times m}, \tag{12}$$

where Γ^+_{ij} and Γ^-_{ij} collect, respectively, the most favourable and least favourable rating recorded for the pair (S_i, ζ_j) across all k DMs:

$$\Gamma^+_{ij} = \left\langle [\mathsf{a}^+_{ij}, \mathsf{b}^+_{ij}], [\mathsf{c}^+_{ij}, \mathsf{d}^+_{ij}] \right\rangle = \begin{cases} \left\langle \left[\max\limits_{1\le r\le k} \mathsf{a}^r_{ij},\ \max\limits_{1\le r\le k} \mathsf{b}^r_{ij}\right],\ \left[\min\limits_{1\le r\le k} \mathsf{c}^r_{ij},\ \min\limits_{1\le r\le k} \mathsf{d}^r_{ij}\right] \right\rangle, & \text{if } \zeta_j \in \text{Benefit},\\ \left\langle \left[\min\limits_{1\le r\le k} \mathsf{a}^r_{ij},\ \min\limits_{1\le r\le k} \mathsf{b}^r_{ij}\right],\ \left[\max\limits_{1\le r\le k} \mathsf{c}^r_{ij},\ \max\limits_{1\le r\le k} \mathsf{d}^r_{ij}\right] \right\rangle, & \text{if } \zeta_j \in \text{Cost}, \end{cases} \tag{13}$$

$$\Gamma^-_{ij} = \left\langle [\mathsf{a}^-_{ij}, \mathsf{b}^-_{ij}], [\mathsf{c}^-_{ij}, \mathsf{d}^-_{ij}] \right\rangle = \begin{cases} \left\langle \left[\min\limits_{1\le r\le k} \mathsf{a}^r_{ij},\ \min\limits_{1\le r\le k} \mathsf{b}^r_{ij}\right],\ \left[\max\limits_{1\le r\le k} \mathsf{c}^r_{ij},\ \max\limits_{1\le r\le k} \mathsf{d}^r_{ij}\right] \right\rangle, & \text{if } \zeta_j \in \text{Benefit},\\ \left\langle \left[\max\limits_{1\le r\le k} \mathsf{a}^r_{ij},\ \max\limits_{1\le r\le k} \mathsf{b}^r_{ij}\right],\ \left[\min\limits_{1\le r\le k} \mathsf{c}^r_{ij},\ \min\limits_{1\le r\le k} \mathsf{d}^r_{ij}\right] \right\rangle, & \text{if } \zeta_j \in \text{Cost}. \end{cases} \tag{14}$$

Applying Definition 6 to average, respectively, over the k DMs and over the n alternatives yields the mean rating $\bar{\Gamma}_{ij}$ of each DM, together with the mean UOA profile $\bar{\Gamma}^+_j$ and mean LOA profile $\bar{\Gamma}^-_j$ for every attribute ζ_j:

$$\bar{\Gamma}_{ij} = \big\langle [\bar{\mathsf{a}}_{ij}, \bar{\mathsf{b}}_{ij}], [\bar{\mathsf{c}}_{ij}, \bar{\mathsf{d}}_{ij}] \big\rangle = \left\langle \left[\frac{1}{k}\sum_{r=1}^{k} \mathsf{a}_{ij}^{r},\ \frac{1}{k}\sum_{r=1}^{k} \mathsf{b}_{ij}^{r} \right], \left[\frac{1}{k}\sum_{r=1}^{k} \mathsf{c}_{ij}^{r},\ \frac{1}{k}\sum_{r=1}^{k} \mathsf{d}_{ij}^{r} \right] \right\rangle, \tag{15}$$

$$\bar{\Gamma}_{j}^{+} = \left\langle \left[\frac{1}{n}\sum_{i=1}^{n} \mathsf{a}_{ij}^{+},\ \frac{1}{n}\sum_{i=1}^{n} \mathsf{b}_{ij}^{+} \right], \left[\frac{1}{n}\sum_{i=1}^{n} \mathsf{c}_{ij}^{+},\ \frac{1}{n}\sum_{i=1}^{n} \mathsf{d}_{ij}^{+} \right] \right\rangle, \tag{16}$$

$$\bar{\Gamma}_{j}^{-} = \left\langle \left[\frac{1}{n}\sum_{i=1}^{n} \mathsf{a}_{ij}^{-},\ \frac{1}{n}\sum_{i=1}^{n} \mathsf{b}_{ij}^{-} \right], \left[\frac{1}{n}\sum_{i=1}^{n} \mathsf{c}_{ij}^{-},\ \frac{1}{n}\sum_{i=1}^{n} \mathsf{d}_{ij}^{-} \right] \right\rangle. \tag{17}$$

For every alternative S_i, treating the m attributes as the "elements" over which Definition 9 operates, the correlation of the rth DM's rating profile with the UOA profile is

$$R\big(\Gamma_i^r, \Gamma_i^+\big) = \frac{C\big(\Gamma_{ij}^r, \Gamma_{ij}^+\big)}{\sqrt{V\big(\Gamma_{ij}^r\big)V\big(\Gamma_{ij}^+\big)}}$$
$$= \frac{\dfrac{1}{2(m-1)}\displaystyle\sum_{j=1}^{m}\left(\begin{array}{c} \Big((\mathsf{a}_{ij}^r)^p - (\bar{\mathsf{a}}_{ij})^p\Big)\Big((\mathsf{a}_{ij}^+)^p - (\bar{\mathsf{a}}_j^+)^p\Big) + \Big((\mathsf{b}_{ij}^r)^p - (\bar{\mathsf{b}}_{ij})^p\Big)\Big((\mathsf{b}_{ij}^+)^p - (\bar{\mathsf{b}}_j^+)^p\Big) \\ +\Big((\mathsf{c}_{ij}^r)^q - (\bar{\mathsf{c}}_{ij})^q\Big)\Big((\mathsf{c}_{ij}^+)^q - (\bar{\mathsf{c}}_j^+)^q\Big) + \Big((\mathsf{d}_{ij}^r)^q - (\bar{\mathsf{d}}_{ij})^q\Big)\Big((\mathsf{d}_{ij}^+)^q - (\bar{\mathsf{d}}_j^+)^q\Big) \end{array} \right)}{\sqrt{\dfrac{1}{2(m-1)}\displaystyle\sum_{j=1}^{m}\left(\begin{array}{c} \Big((\mathsf{a}_{ij}^r)^p - (\bar{\mathsf{a}}_{ij})^p\Big)^2 + \\ \Big((\mathsf{b}_{ij}^r)^p - (\bar{\mathsf{b}}_{ij})^p\Big)^2 + \\ \Big((\mathsf{c}_{ij}^r)^q - (\bar{\mathsf{c}}_{ij})^q\Big)^2 + \\ \Big((\mathsf{d}_{ij}^r)^q - (\bar{\mathsf{d}}_{ij})^q\Big)^2 \end{array} \right) \times \dfrac{1}{2(m-1)}\displaystyle\sum_{j=1}^{m}\left(\begin{array}{c} \Big((\mathsf{a}_{ij}^+)^p - (\bar{\mathsf{a}}_j^+)^p\Big)^2 + \\ \Big((\mathsf{b}_{ij}^+)^p - (\bar{\mathsf{b}}_j^+)^p\Big)^2 + \\ \Big((\mathsf{c}_{ij}^+)^q - (\bar{\mathsf{c}}_j^+)^q\Big)^2 + \\ \Big((\mathsf{d}_{ij}^+)^q - (\bar{\mathsf{d}}_j^+)^q\Big)^2 \end{array} \right)}}. \tag{18}$$

and its correlation with the LOA profile is obtained in the same way, with every "$+$" superscript replaced by a "$-$" superscript:

$$R\big(\Gamma_i^r, \Gamma_i^-\big) = \frac{C\big(\Gamma_{ij}^r, \Gamma_{ij}^-\big)}{\sqrt{V\big(\Gamma_{ij}^r\big)V\big(\Gamma_{ij}^-\big)}}$$
$$= \frac{\dfrac{1}{2(m-1)}\displaystyle\sum_{j=1}^{m}\left(\begin{array}{c} \Big((\mathsf{a}_{ij}^r)^p - (\bar{\mathsf{a}}_{ij})^p\Big)\Big((\mathsf{a}_{ij}^-)^p - (\bar{\mathsf{a}}_j^-)^p\Big) + \Big((\mathsf{b}_{ij}^r)^p - (\bar{\mathsf{b}}_{ij})^p\Big)\Big((\mathsf{b}_{ij}^-)^p - (\bar{\mathsf{b}}_j^-)^p\Big) \\ +\Big((\mathsf{c}_{ij}^r)^q - (\bar{\mathsf{c}}_{ij})^q\Big)\Big((\mathsf{c}_{ij}^-)^q - (\bar{\mathsf{c}}_j^-)^q\Big) + \Big((\mathsf{d}_{ij}^r)^q - (\bar{\mathsf{d}}_{ij})^q\Big)\Big((\mathsf{d}_{ij}^-)^q - (\bar{\mathsf{d}}_j^-)^q\Big) \end{array} \right)}{\sqrt{\dfrac{1}{2(m-1)}\displaystyle\sum_{j=1}^{m}\left(\begin{array}{c} \Big((\mathsf{a}_{ij}^r)^p - (\bar{\mathsf{a}}_{ij})^p\Big)^2 + \\ \Big((\mathsf{b}_{ij}^r)^p - (\bar{\mathsf{b}}_{ij})^p\Big)^2 + \\ \Big((\mathsf{c}_{ij}^r)^q - (\bar{\mathsf{c}}_{ij})^q\Big)^2 + \\ \Big((\mathsf{d}_{ij}^r)^q - (\bar{\mathsf{d}}_{ij})^q\Big)^2 \end{array} \right) \times \dfrac{1}{2(m-1)}\displaystyle\sum_{j=1}^{m}\left(\begin{array}{c} \Big((\mathsf{a}_{ij}^-)^p - (\bar{\mathsf{a}}_j^-)^p\Big)^2 + \\ \Big((\mathsf{b}_{ij}^-)^p - (\bar{\mathsf{b}}_j^-)^p\Big)^2 + \\ \Big((\mathsf{c}_{ij}^-)^q - (\bar{\mathsf{c}}_j^-)^q\Big)^2 + \\ \Big((\mathsf{d}_{ij}^-)^q - (\bar{\mathsf{d}}_j^-)^q\Big)^2 \end{array} \right)}}. \tag{19}$$

Averaging the magnitudes of these correlations over all n alternatives gives the overall alignment of the rth DM with the UOA and LOA,

$$R^{r+} = \frac{1}{n}\sum_{i=1}^{n}\big|R(\Gamma_i^r, \Gamma_i^+)\big|, \qquad R^{r-} = \frac{1}{n}\sum_{i=1}^{n}\big|R(\Gamma_i^r, \Gamma_i^-)\big|. \tag{20}$$

Absolute values are taken here because a DM's correlation with the UOA or the LOA may come out negative, and an unsigned distortion of this kind would otherwise skew the comparison across DMs. What matters for the weighting scheme is only how strongly each DM's ratings align with the two reference profiles, not the sign of that alignment: a DM whose absolute correlation is large is drifting away from the panel's collective judgement (behaving, in effect, as an outlier), whereas one whose absolute correlation stays low is tracking the group consensus more closely and should, on that basis, be trusted more. This reasoning is encoded through the similarity degree

$$S^r = \frac{R^{r-}}{R^{r+} + R^{r-}}, \tag{21}$$

which measures how much closer the rth DM sits to the LOA than to the UOA; a larger S^r signals a rating pattern that is nearer to the panel consensus. Normalizing these degrees across all k DMs then delivers the DM weight vector,

$$\delta^r = \frac{S^r}{\sum\limits_{r=1}^{k} S^r}, \tag{22}$$

where $0 \leq \delta^r \leq 1$ and $\sum\limits_{r=1}^{k} \delta^r = 1$.

Once the weights δ^r have been fixed, the k individual decision matrices are fused into a single consensus matrix $D = (\Gamma_{ij})_{n\times m}$ by invoking the IVpq-ROFWA operator of Definition 5,

$$\Gamma_{ij} = \text{IV}pq\text{-ROFWA}\big(\Gamma_{ij}^1, \Gamma_{ij}^2, \ldots, \Gamma_{ij}^k\big) = \bigoplus_{r=1}^{k} \delta^r \Gamma_{ij}^r = \big\langle [\mathsf{a}_{ij}, \mathsf{b}_{ij}], [\mathsf{c}_{ij}, \mathsf{d}_{ij}] \big\rangle$$
$$= \left\langle \left[\sqrt[p]{1 - \prod_{r=1}^{k} \big(1 - (\mathsf{a}_{ij}^r)^p\big)^{\delta^r}},\ \sqrt[p]{1 - \prod_{r=1}^{k} \big(1 - (\mathsf{b}_{ij}^r)^p\big)^{\delta^r}} \right], \left[\prod_{r=1}^{k} \big(\mathsf{c}_{ij}^r\big)^{\delta^r}, \prod_{r=1}^{k} \big(\mathsf{d}_{ij}^r\big)^{\delta^r} \right] \right\rangle. \tag{23}$$

Finally, so that cost- and benefit-type attributes no longer pull the aggregated matrix in opposite directions, $D = (\Gamma_{ij})_{n\times m}$ is passed through the normalization rule

$$\widetilde{\Gamma}_{ij} = \big\langle [\widetilde{\mathsf{a}}_{ij}, \widetilde{\mathsf{b}}_{ij}], [\widetilde{\mathsf{c}}_{ij}, \widetilde{\mathsf{d}}_{ij}] \big\rangle = \begin{cases} \big\langle [\mathsf{a}_{ij}, \mathsf{b}_{ij}], [\mathsf{c}_{ij}, \mathsf{d}_{ij}] \big\rangle, & \text{if } \zeta_j \in \text{Benefit}, \\ \big\langle [\mathsf{c}_{ij}, \mathsf{d}_{ij}], [\mathsf{a}_{ij}, \mathsf{b}_{ij}] \big\rangle, & \text{if } \zeta_j \in \text{Cost}, \end{cases} \tag{24}$$

producing the normalized matrix $\widetilde{D} = (\widetilde{\Gamma}_{ij})_{n\times m}$ that is carried forward into the ranking stage.

4.2 Ranking via linear programming

Once the k individual matrices $D^r = (\Gamma_{ij}^r)_{n\times m}$ have been fused, through the IVpq-ROFWA operator, into the single consensus matrix $D = (\Gamma_{ij})_{n\times m}$, this matrix becomes the starting point for ranking the alternatives $S_1, \ldots, S_n$. Rather than fixing one common set of attribute weights for the whole panel, we solve, for every alternative S_i in turn, a dedicated optimization problem that asks how well S_i can perform under the weighting most favourable to it — a construction borrowed from the logic of data envelopment analysis (DEA), where each unit under evaluation is allowed its own best-case weight profile. To keep this freedom from inflating scores artificially, every model still requires that no alternative, including the others being compared, fall below a common minimum acceptability profile Γ^0. Working on the normalized matrix $\widetilde{D} = (\widetilde{\Gamma}_{ij})_{n\times m}$ obtained from Eq. (24), the LP model attached to alternative S_i reads

$$\text{Maximize} \quad \sum_{j=1}^{m} \widetilde{\Gamma}_{ij}\, y_j \tag{25}$$

$$\text{Subject to:} \quad \sum_{j=1}^{m} \widetilde{\Gamma}_{kj}\, y_j \succeq \Gamma^0, \quad k = 1, 2, \ldots, n, \qquad y_j \geq 0,\ j = 1, 2, \ldots, m, \tag{26}$$

where "$\succeq$" denotes the ranking order of Definition 4. Here the y_j act as free, non-negative attribute weights, and the objective aggregates the attribute-wise IVpq-ROFNs $\widetilde{\Gamma}_{ij}$ into one composite fuzzy score for S_i; maximizing it identifies the most favourable outcome S_i can attain, while the n constraints require every alternative, evaluated under that same weighting, to remain above the threshold Γ^0.

Remark 2. *The choice of the threshold Γ^0 is not incidental, since it shapes both the feasibility and the interpretation of model* (25)–(26)*. Using the score function of Definition 3, an analyst can check whether the score attached to Γ^0 already meets the acceptability level they have in mind, adjusting Γ^0 upward otherwise. In this study, an alternative is deemed acceptable once its score exceeds 0.50, i.e., once it captures at least half of the maximum attainable fuzzy performance, with larger scores marking stronger alternatives.*

Rationale for individual alternative-based optimization

Solving a separate LP for each alternative, instead of imposing one shared weight vector on the whole panel, lets every alternative be judged on the profile of attributes where it genuinely excels rather than forcing all of them through the same lens, a property that matters when alternatives differ markedly in their strengths, as is typical in sustainable-supplier type problems. This mirrors the DEA principle of granting each decision-making unit its own best-case weighting, an idea that similar fuzzy LP-based studies have also found effective for interval-valued and hesitant assessments. Beyond ranking, the formulation doubles as a filter: alternatives unable to clear the threshold Γ^0 under any weighting are effectively screened out, while the remaining ones are ordered by their optimized scores; these rankings are later cross-checked against TOPSIS and Copeland fusion for consistency.

To make model (25)–(26) tractable, we rescale the weights so that they sum to one, via

$$t = \frac{1}{\sum_{j=1}^{m} y_j} \quad \Longrightarrow \quad \sum_{j=1}^{m} y_j = \frac{1}{t}, \tag{27}$$

and, setting $z_j = t\, y_j$ for $j = 1, \ldots, m$, model (25)–(26) becomes

$$\text{Maximize} \quad \sum_{j=1}^{m} \widetilde{\Gamma}_{ij}\, z_j \tag{28}$$

$$\text{Subject to:} \quad \sum_{j=1}^{m} \widetilde{\Gamma}_{ij}\, z_j \succeq t\,\Gamma^0, \quad i = 1, 2, \ldots, n, \qquad \sum_{j=1}^{m} z_j = 1, \quad z_j \geq 0,\ j = 1, \ldots, m, \quad t \geq \epsilon > 0. \tag{29}$$

Because $\sum\limits_{j=1}^{m} z_j = 1$ with $z_j \geq 0$, the aggregation on the left of Eq. (28) is exactly an IVpq-ROFWA combination of $\widetilde{\Gamma}_{i1}, \ldots, \widetilde{\Gamma}_{im}$ with weights z_j, so that, following Definition 5,

$$\sum_{j=1}^{m} \widetilde{\Gamma}_{ij}\, z_j = \left\langle \left[\sqrt[p]{1 - \prod_{j=1}^{m} \left(1 - (\widetilde{\mathsf{a}}_{ij})^p\right)^{z_j}},\ \sqrt[p]{1 - \prod_{j=1}^{m} \left(1 - (\widetilde{\mathsf{b}}_{ij})^p\right)^{z_j}} \right],\ \left[\prod_{j=1}^{m} (\widetilde{\mathsf{c}}_{ij})^{z_j},\ \prod_{j=1}^{m} (\widetilde{\mathsf{d}}_{ij})^{z_j} \right] \right\rangle. \tag{30}$$

By Definition 3, an IVpq-ROFN scores higher precisely when its membership component grows and its non-membership component shrinks. Eq. (30) therefore attains its best value exactly when

$$1 - \prod_{j=1}^{m} \left(1 - (\widetilde{\mathsf{a}}_{ij})^p\right)^{z_j} \text{ and } 1 - \prod_{j=1}^{m} \left(1 - (\widetilde{\mathsf{b}}_{ij})^p\right)^{z_j}$$

are pushed up while

$$\prod_{j=1}^{m}(\widetilde{\mathsf{c}}_{ij})^{z_j} \quad \text{and} \quad \prod_{j=1}^{m}(\widetilde{\mathsf{d}}_{ij})^{z_j}$$

are pulled down — equivalently, once the following four quantities are simultaneously minimized,

$$\prod_{j=1}^{m}\left(1-(\widetilde{\mathsf{a}}_{ij})^p\right)^{z_j}, \quad \prod_{j=1}^{m}\left(1-(\widetilde{\mathsf{b}}_{ij})^p\right)^{z_j}, \quad \prod_{j=1}^{m}(\widetilde{\mathsf{c}}_{ij})^{z_j}, \quad \prod_{j=1}^{m}(\widetilde{\mathsf{d}}_{ij})^{z_j}. \tag{31}$$

Model (28)–(29) thus reduces to the multi-objective problem

$$\text{Minimize} \quad \left(\prod_{j=1}^{m}\left(1-(\widetilde{\mathsf{a}}_{ij})^p\right)^{z_j}, \prod_{j=1}^{m}\left(1-(\widetilde{\mathsf{b}}_{ij})^p\right)^{z_j}, \prod_{j=1}^{m}(\widetilde{\mathsf{c}}_{ij})^{z_j}, \prod_{j=1}^{m}(\widetilde{\mathsf{d}}_{ij})^{z_j}\right). \tag{32}$$

Since the natural logarithm is strictly increasing, minimizing each term in Eq. (32) is equivalent to minimizing its logarithm,

$$\text{Minimize} \quad \left(\sum_{j=1}^{m} z_j \ln\left(1-(\widetilde{\mathsf{a}}_{ij})^p\right), \sum_{j=1}^{m} z_j \ln\left(1-(\widetilde{\mathsf{b}}_{ij})^p\right), \sum_{j=1}^{m} z_j \ln(\widetilde{\mathsf{c}}_{ij}), \sum_{j=1}^{m} z_j \ln(\widetilde{\mathsf{d}}_{ij})\right), \tag{33}$$

and, all four quantities lying in $[0,1]$, they can be folded into one scalar objective by summing them:

$$\text{Minimize} \quad \sum_{j=1}^{m} z_j \ln\left(\left(1-(\widetilde{\mathsf{a}}_{ij})^p\right)\left(1-(\widetilde{\mathsf{b}}_{ij})^p\right)\widetilde{\mathsf{c}}_{ij}\,\widetilde{\mathsf{d}}_{ij}\right). \tag{34}$$

Turning next to the ith constraint in Eq. (29), its left-hand side expands, via Eq. (30), to

$$\left\langle \left[\sqrt[p]{1-\prod_{j=1}^{m}\left(1-(\widetilde{\mathsf{a}}_{ij})^p\right)^{z_j}}, \sqrt[p]{1-\prod_{j=1}^{m}\left(1-(\widetilde{\mathsf{b}}_{ij})^p\right)^{z_j}}\right], \left[\prod_{j=1}^{m}(\widetilde{\mathsf{c}}_{ij})^{z_j}, \prod_{j=1}^{m}(\widetilde{\mathsf{d}}_{ij})^{z_j}\right]\right\rangle, \tag{35}$$

while its right-hand side, $t\,\Gamma^0$ with $\Gamma^0 = \langle [a_0, b_0], [c_0, d_0] \rangle$, follows from the scalar-multiplication rule of Definition 2 as

$$\left\langle \left[\left(1-(1-a_0^p)^t\right)^{1/p}, \left(1-(1-b_0^p)^t\right)^{1/p}\right], \left[c_0^t, d_0^t\right]\right\rangle. \tag{36}$$

Comparing membership (before extracting the pth root) and non-membership degrees between Eqs. (35) and (36) turns the constraint into

$$\begin{cases} 1-\prod_{j=1}^{m}\left(1-(\widetilde{\mathsf{a}}_{ij})^p\right)^{z_j} \geq 1-(1-a_0^p)^t, \\ 1-\prod_{j=1}^{m}\left(1-(\widetilde{\mathsf{b}}_{ij})^p\right)^{z_j} \geq 1-(1-b_0^p)^t, \\ \prod_{j=1}^{m}(\widetilde{\mathsf{c}}_{ij})^{z_j} \leq c_0^t, \\ \prod_{j=1}^{m}(\widetilde{\mathsf{d}}_{ij})^{z_j} \leq d_0^t, \end{cases} \tag{37}$$

and, since $\ln(\cdot)$ preserves inequality direction, Eq. (37) linearizes to

$$\begin{cases} \sum_{j=1}^{m} z_j \ln\left(1-(\widetilde{\mathsf{a}}_{ij})^p\right) \leq t \ln\left(1-a_0^p\right), \\ \sum_{j=1}^{m} z_j \ln\left(1-(\widetilde{\mathsf{b}}_{ij})^p\right) \leq t \ln\left(1-b_0^p\right), \\ \sum_{j=1}^{m} z_j \ln(\widetilde{\mathsf{c}}_{ij}) \leq t \ln(c_0), \\ \sum_{j=1}^{m} z_j \ln(\widetilde{\mathsf{d}}_{ij}) \leq t \ln(d_0). \end{cases} \tag{38}$$

Collecting Eqs. (34) and (38), the IVpq-ROF linear programming model for alternative S_i takes its final, solvable form:

$$\text{Minimize} \quad \sum_{j=1}^{m} z_j \ln\left(\left(1-(\widetilde{\mathsf{a}}_{ij})^p\right)\left(1-(\widetilde{\mathsf{b}}_{ij})^p\right)\widetilde{\mathsf{c}}_{ij}\,\widetilde{\mathsf{d}}_{ij}\right) \tag{39}$$

$$\text{Subject to:} \begin{cases} \sum\limits_{j=1}^{m} z_j \ln\left(1-(\widetilde{\mathsf{a}}_{ij})^p\right) \leq t \ln\left(1-a_0^p\right), \\ \sum\limits_{j=1}^{m} z_j \ln\left(1-(\widetilde{\mathsf{b}}_{ij})^p\right) \leq t \ln\left(1-b_0^p\right), \\ \sum\limits_{j=1}^{m} z_j \ln(\widetilde{\mathsf{c}}_{ij}) \leq t \ln(c_0), \\ \sum\limits_{j=1}^{m} z_j \ln(\widetilde{\mathsf{d}}_{ij}) \leq t \ln(d_0), \\ \sum\limits_{j=1}^{m} z_j = 1, \quad z_j \geq 0,\ j=1,\ldots,m, \quad t \geq \epsilon > 0. \end{cases} \tag{40}$$

Model (39)–(40) is a genuine linear program, solvable with any standard simplex or interior-point routine, and is run once for each of the n alternatives. Because the transformation traded a maximization for a minimization of the log-objective, a *lower* optimal value now signals a *more* preferable alternative, so the n resulting scores are finally ranked in ascending order.

5. Illustration

Access to timely and reliable healthcare remains a persistent challenge in rural settings, where distance, limited specialist availability, and thin infrastructure often stand between patients and adequate care. Digital telehealth platforms have emerged as a practical response to this gap, allowing remote consultation, diagnosis, and follow-up care to be delivered without requiring patients to travel to distant facilities. Choosing among the many platforms now on offer, however, is far from straightforward: a platform judged excellent on clinical grounds may prove impractical where bandwidth is scarce, while one that is technically robust may still be too costly for a resource-constrained health authority to sustain. Such a decision therefore has to weigh clinical, technical, and economic considerations together, under conditions where the evidence available to decision-makers is rarely precise or complete.

To illustrate the proposed IVpq-ROF framework, we consider a health authority tasked with selecting a telehealth platform for deployment across a network of rural clinics. Eight candidate platforms, denoted $P_1, P_2, \ldots, P_8$, are shortlisted for evaluation. Three domain experts (DMs) take part in the assessment: DM_1 is a clinical informatics specialist familiar with remote-diagnosis workflows; DM_2 is a health-IT infrastructure engineer with expertise in system interoperability and network performance; and DM_3 is a healthcare finance and operations manager responsible for budgeting and long-term sustainability of the deployment. Each platform is judged against ten attributes spanning three dimensions, clinical/service quality, technical infrastructure, and economic/operational feasibility, as summarised in Table 1.

Every judgement is recorded as an IVpq-ROFN, in the manner of Definition 1: an entry such as $\langle[0.5, 0.75], [0.25, 0.35]\rangle$ indicates that, in the expert's assessment, a platform meets a given criterion with a membership degree somewhere in $[0.5, 0.75]$ and a non-membership degree somewhere in $[0.25, 0.35]$. Allowing the two grades their own exponents, rather than forcing them to share one, widens the region in which such judgements remain admissible; for the data considered in this illustration, the smallest rung parameters for which the orthopair constraint holds throughout are $p = 2$ and $q = 3$, and these values are adopted for the remainder of this study.

Table 1
Description of evaluation attributes

Dimension	Notation	Attribute description
Clinical / Service	ζ_1	Accuracy and adequacy of remote diagnostic and monitoring support offered to clinicians.
	ζ_2	Strength of patient-data privacy safeguards and regulatory compliance.
	ζ_3	Ease of scheduling consultations and overall accessibility for patients.
	ζ_4	Availability of multilingual and accessibility features suited to diverse rural populations.
Technical	ζ_5	Ability to function reliably under low-bandwidth and intermittent connectivity conditions.
	ζ_6	Ease of integration with existing hospital information and records systems.
	ζ_7	Overall system reliability and service uptime.
Economic / Operational	ζ_8	Deployment and recurring subscription cost (cost-type attribute).
	ζ_9	Extent of staff training required for adoption (cost-type attribute).
	ζ_{10}	Scalability of the platform to additional rural centres over time.

Figure 1 outlines the overall solution framework adopted in this study, tracing the path from data collection through DM and attribute weighting, LP-based ranking, and cross-validation via CRITIC–TOPSIS and Copeland rank fusion.

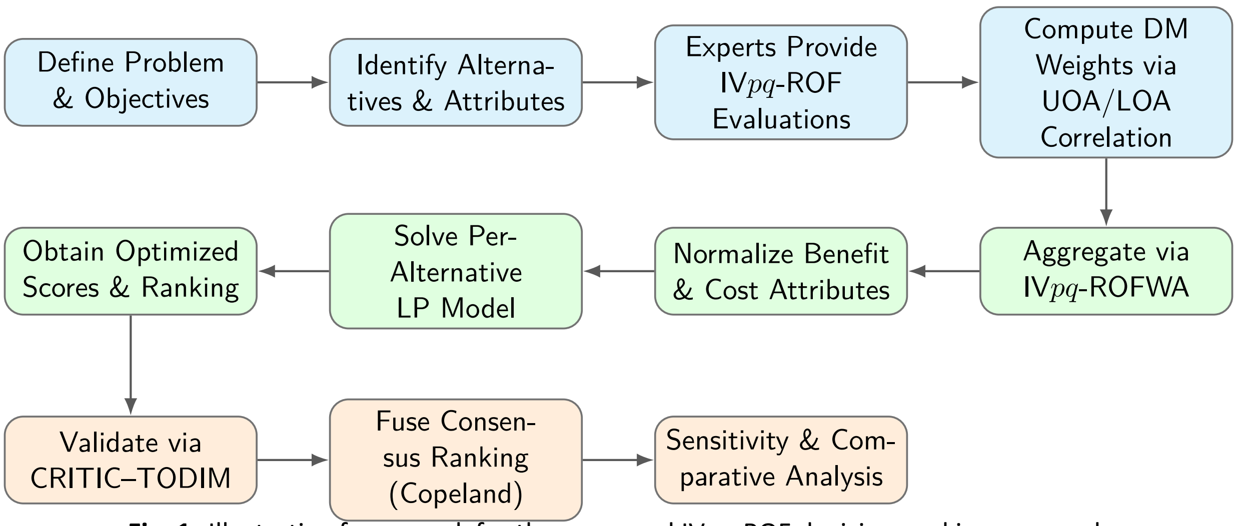

Fig. 1. Illustrative framework for the proposed IVpq-ROF decision-making approach

5.1 Solution

The steps of the proposed IVpq-ROF-based platform-selection methodology are outlined below. Tables 2 and 3 present the raw judgements supplied by the three DMs.

Table 2
Assessment from DM_r (ζ_1–ζ_5)

DM_r	Alt.	ζ_1	ζ_2	ζ_3
DM_1	P_1	$\langle[0.55,0.90],[0.55,0.57]\rangle$	$\langle[0.54,0.62],[0.25,0.36]\rangle$	$\langle[0.61,0.81],[0.46,0.61]\rangle$
	P_2	$\langle[0.52,0.67],[0.72,0.74]\rangle$	$\langle[0.43,0.54],[0.18,0.27]\rangle$	$\langle[0.75,0.95],[0.36,0.40]\rangle$
	P_3	$\langle[0.56,0.73],[0.45,0.51]\rangle$	$\langle[0.66,0.75],[0.29,0.49]\rangle$	$\langle[0.39,0.48],[0.47,0.52]\rangle$
	P_4	$\langle[0.36,0.53],[0.23,0.37]\rangle$	$\langle[0.36,0.45],[0.52,0.66]\rangle$	$\langle[0.57,0.64],[0.56,0.75]\rangle$
	P_5	$\langle[0.52,0.64],[0.42,0.58]\rangle$	$\langle[0.64,0.84],[0.38,0.51]\rangle$	$\langle[0.56,0.75],[0.57,0.69]\rangle$
	P_6	$\langle[0.48,0.63],[0.41,0.45]\rangle$	$\langle[0.57,0.73],[0.42,0.48]\rangle$	$\langle[0.52,0.55],[0.42,0.48]\rangle$
	P_7	$\langle[0.48,0.61],[0.25,0.36]\rangle$	$\langle[0.43,0.50],[0.45,0.56]\rangle$	$\langle[0.53,0.62],[0.40,0.59]\rangle$
	P_8	$\langle[0.42,0.60],[0.46,0.53]\rangle$	$\langle[0.44,0.55],[0.36,0.44]\rangle$	$\langle[0.39,0.43],[0.30,0.40]\rangle$
DM_2	P_1	$\langle[0.40,0.55],[0.28,0.44]\rangle$	$\langle[0.53,0.67],[0.36,0.48]\rangle$	$\langle[0.56,0.63],[0.59,0.68]\rangle$
	P_2	$\langle[0.49,0.65],[0.53,0.71]\rangle$	$\langle[0.51,0.56],[0.33,0.44]\rangle$	$\langle[0.54,0.67],[0.25,0.36]\rangle$
	P_3	$\langle[0.50,0.58],[0.51,0.63]\rangle$	$\langle[0.71,0.84],[0.26,0.44]\rangle$	$\langle[0.39,0.45],[0.42,0.53]\rangle$
	P_4	$\langle[0.43,0.54],[0.36,0.46]\rangle$	$\langle[0.30,0.36],[0.38,0.52]\rangle$	$\langle[0.55,0.90],[0.55,0.57]\rangle$
	P_5	$\langle[0.60,0.74],[0.47,0.50]\rangle$	$\langle[0.69,0.81],[0.48,0.58]\rangle$	$\langle[0.59,0.70],[0.51,0.64]\rangle$
	P_6	$\langle[0.52,0.57],[0.49,0.63]\rangle$	$\langle[0.40,0.47],[0.30,0.39]\rangle$	$\langle[0.66,0.77],[0.21,0.34]\rangle$
	P_7	$\langle[0.54,0.67],[0.31,0.45]\rangle$	$\langle[0.47,0.54],[0.45,0.60]\rangle$	$\langle[0.62,0.78],[0.30,0.42]\rangle$
	P_8	$\langle[0.44,0.62],[0.37,0.53]\rangle$	$\langle[0.43,0.52],[0.40,0.55]\rangle$	$\langle[0.58,0.77],[0.48,0.63]\rangle$
DM_3	P_1	$\langle[0.43,0.53],[0.33,0.55]\rangle$	$\langle[0.72,0.77],[0.41,0.54]\rangle$	$\langle[0.63,0.71],[0.52,0.62]\rangle$
	P_2	$\langle[0.57,0.72],[0.49,0.62]\rangle$	$\langle[0.58,0.65],[0.19,0.30]\rangle$	$\langle[0.64,0.77],[0.33,0.53]\rangle$
	P_3	$\langle[0.52,0.65],[0.30,0.38]\rangle$	$\langle[0.60,0.78],[0.27,0.31]\rangle$	$\langle[0.38,0.52],[0.44,0.58]\rangle$
	P_4	$\langle[0.49,0.58],[0.26,0.37]\rangle$	$\langle[0.38,0.53],[0.37,0.41]\rangle$	$\langle[0.65,0.78],[0.41,0.63]\rangle$
	P_5	$\langle[0.52,0.68],[0.40,0.52]\rangle$	$\langle[0.59,0.67],[0.42,0.56]\rangle$	$\langle[0.54,0.64],[0.52,0.69]\rangle$
	P_6	$\langle[0.52,0.67],[0.53,0.66]\rangle$	$\langle[0.39,0.48],[0.44,0.55]\rangle$	$\langle[0.65,0.79],[0.38,0.45]\rangle$
	P_7	$\langle[0.55,0.74],[0.28,0.47]\rangle$	$\langle[0.39,0.47],[0.36,0.49]\rangle$	$\langle[0.62,0.75],[0.31,0.44]\rangle$
	P_8	$\langle[0.30,0.49],[0.31,0.44]\rangle$	$\langle[0.32,0.45],[0.47,0.62]\rangle$	$\langle[0.50,0.65],[0.41,0.56]\rangle$

Table 2
Continued

DM_r	Alt.	ζ_4	ζ_5
DM_1	P_1	$\langle[0.57,0.70],[0.42,0.62]\rangle$	$\langle[0.28,0.43],[0.32,0.45]\rangle$
	P_2	$\langle[0.51,0.64],[0.39,0.54]\rangle$	$\langle[0.42,0.47],[0.53,0.61]\rangle$
	P_3	$\langle[0.35,0.53],[0.33,0.46]\rangle$	$\langle[0.58,0.82],[0.39,0.45]\rangle$
	P_4	$\langle[0.61,0.77],[0.45,0.52]\rangle$	$\langle[0.41,0.48],[0.43,0.52]\rangle$
	P_5	$\langle[0.73,0.82],[0.39,0.53]\rangle$	$\langle[0.70,0.82],[0.44,0.54]\rangle$
	P_6	$\langle[0.29,0.42],[0.25,0.38]\rangle$	$\langle[0.71,0.77],[0.43,0.53]\rangle$
	P_7	$\langle[0.58,0.70],[0.29,0.47]\rangle$	$\langle[0.49,0.51],[0.44,0.46]\rangle$
	P_8	$\langle[0.48,0.65],[0.23,0.29]\rangle$	$\langle[0.56,0.67],[0.38,0.58]\rangle$
DM_2	P_1	$\langle[0.58,0.67],[0.56,0.65]\rangle$	$\langle[0.28,0.38],[0.37,0.50]\rangle$
	P_2	$\langle[0.38,0.49],[0.33,0.40]\rangle$	$\langle[0.35,0.51],[0.49,0.54]\rangle$
	P_3	$\langle[0.35,0.47],[0.28,0.46]\rangle$	$\langle[0.55,0.90],[0.55,0.57]\rangle$
	P_4	$\langle[0.55,0.58],[0.43,0.63]\rangle$	$\langle[0.57,0.69],[0.47,0.55]\rangle$
	P_5	$\langle[0.56,0.70],[0.31,0.50]\rangle$	$\langle[0.62,0.73],[0.55,0.73]\rangle$
	P_6	$\langle[0.45,0.63],[0.30,0.37]\rangle$	$\langle[0.58,0.73],[0.37,0.49]\rangle$
	P_7	$\langle[0.41,0.47],[0.31,0.46]\rangle$	$\langle[0.34,0.44],[0.49,0.58]\rangle$
	P_8	$\langle[0.47,0.60],[0.28,0.39]\rangle$	$\langle[0.45,0.65],[0.39,0.53]\rangle$
DM_3	P_1	$\langle[0.58,0.74],[0.39,0.50]\rangle$	$\langle[0.29,0.39],[0.42,0.55]\rangle$
	P_2	$\langle[0.34,0.50],[0.49,0.73]\rangle$	$\langle[0.35,0.46],[0.38,0.52]\rangle$
	P_3	$\langle[0.53,0.68],[0.34,0.46]\rangle$	$\langle[0.64,0.83],[0.43,0.47]\rangle$
	P_4	$\langle[0.65,0.76],[0.50,0.56]\rangle$	$\langle[0.43,0.55],[0.43,0.58]\rangle$
	P_5	$\langle[0.61,0.81],[0.27,0.45]\rangle$	$\langle[0.69,0.83],[0.55,0.64]\rangle$
	P_6	$\langle[0.39,0.45],[0.26,0.43]\rangle$	$\langle[0.61,0.66],[0.51,0.67]\rangle$
	P_7	$\langle[0.47,0.65],[0.24,0.49]\rangle$	$\langle[0.41,0.57],[0.42,0.59]\rangle$
	P_8	$\langle[0.40,0.45],[0.23,0.35]\rangle$	$\langle[0.57,0.66],[0.41,0.57]\rangle$

Table 3
Assessment from DM_r (ζ_6–ζ_{10})

DM_r	Alt.	ζ_6	ζ_7	ζ_8
DM_1	P_1	$\langle[0.48, 0.73], [0.21, 0.29]\rangle$	$\langle[0.55, 0.60], [0.53, 0.65]\rangle$	$\langle[0.57, 0.70], [0.58, 0.64]\rangle$
	P_2	$\langle[0.41, 0.54], [0.28, 0.42]\rangle$	$\langle[0.46, 0.57], [0.50, 0.65]\rangle$	$\langle[0.62, 0.76], [0.59, 0.71]\rangle$
	P_3	$\langle[0.51, 0.61], [0.37, 0.43]\rangle$	$\langle[0.35, 0.51], [0.46, 0.58]\rangle$	$\langle[0.56, 0.62], [0.44, 0.60]\rangle$
	P_4	$\langle[0.64, 0.73], [0.37, 0.53]\rangle$	$\langle[0.37, 0.42], [0.45, 0.55]\rangle$	$\langle[0.53, 0.63], [0.42, 0.58]\rangle$
	P_5	$\langle[0.50, 0.52], [0.52, 0.64]\rangle$	$\langle[0.42, 0.57], [0.46, 0.68]\rangle$	$\langle[0.60, 0.73], [0.37, 0.44]\rangle$
	P_6	$\langle[0.46, 0.61], [0.53, 0.68]\rangle$	$\langle[0.52, 0.64], [0.56, 0.68]\rangle$	$\langle[0.53, 0.55], [0.26, 0.46]\rangle$
	P_7	$\langle[0.44, 0.59], [0.48, 0.67]\rangle$	$\langle[0.41, 0.56], [0.43, 0.46]\rangle$	$\langle[0.47, 0.52], [0.36, 0.40]\rangle$
	P_8	$\langle[0.41, 0.62], [0.22, 0.28]\rangle$	$\langle[0.41, 0.55], [0.57, 0.75]\rangle$	$\langle[0.43, 0.56], [0.37, 0.39]\rangle$
DM_2	P_1	$\langle[0.44, 0.61], [0.35, 0.54]\rangle$	$\langle[0.45, 0.53], [0.62, 0.72]\rangle$	$\langle[0.62, 0.76], [0.49, 0.65]\rangle$
	P_2	$\langle[0.38, 0.47], [0.23, 0.30]\rangle$	$\langle[0.45, 0.61], [0.59, 0.74]\rangle$	$\langle[0.68, 0.80], [0.52, 0.66]\rangle$
	P_3	$\langle[0.62, 0.71], [0.29, 0.47]\rangle$	$\langle[0.41, 0.52], [0.55, 0.65]\rangle$	$\langle[0.37, 0.48], [0.42, 0.49]\rangle$
	P_4	$\langle[0.54, 0.71], [0.46, 0.53]\rangle$	$\langle[0.45, 0.59], [0.50, 0.64]\rangle$	$\langle[0.53, 0.67], [0.35, 0.53]\rangle$
	P_5	$\langle[0.49, 0.67], [0.46, 0.50]\rangle$	$\langle[0.44, 0.52], [0.50, 0.61]\rangle$	$\langle[0.60, 0.70], [0.35, 0.52]\rangle$
	P_6	$\langle[0.30, 0.41], [0.55, 0.65]\rangle$	$\langle[0.53, 0.65], [0.49, 0.62]\rangle$	$\langle[0.55, 0.59], [0.12, 0.21]\rangle$
	P_7	$\langle[0.44, 0.61], [0.53, 0.72]\rangle$	$\langle[0.34, 0.42], [0.60, 0.70]\rangle$	$\langle[0.51, 0.71], [0.38, 0.55]\rangle$
	P_8	$\langle[0.39, 0.53], [0.30, 0.36]\rangle$	$\langle[0.34, 0.45], [0.57, 0.68]\rangle$	$\langle[0.55, 0.72], [0.46, 0.63]\rangle$
DM_3	P_1	$\langle[0.45, 0.53], [0.36, 0.45]\rangle$	$\langle[0.42, 0.59], [0.53, 0.63]\rangle$	$\langle[0.60, 0.72], [0.47, 0.53]\rangle$
	P_2	$\langle[0.52, 0.66], [0.32, 0.42]\rangle$	$\langle[0.45, 0.66], [0.50, 0.60]\rangle$	$\langle[0.62, 0.76], [0.51, 0.67]\rangle$
	P_3	$\langle[0.46, 0.58], [0.40, 0.51]\rangle$	$\langle[0.43, 0.59], [0.54, 0.75]\rangle$	$\langle[0.42, 0.59], [0.26, 0.32]\rangle$
	P_4	$\langle[0.60, 0.70], [0.56, 0.77]\rangle$	$\langle[0.39, 0.46], [0.56, 0.71]\rangle$	$\langle[0.52, 0.60], [0.48, 0.65]\rangle$
	P_5	$\langle[0.42, 0.56], [0.50, 0.64]\rangle$	$\langle[0.46, 0.54], [0.36, 0.51]\rangle$	$\langle[0.60, 0.68], [0.28, 0.42]\rangle$
	P_6	$\langle[0.35, 0.44], [0.48, 0.51]\rangle$	$\langle[0.51, 0.70], [0.57, 0.61]\rangle$	$\langle[0.55, 0.90], [0.55, 0.57]\rangle$
	P_7	$\langle[0.44, 0.49], [0.50, 0.65]\rangle$	$\langle[0.40, 0.59], [0.39, 0.58]\rangle$	$\langle[0.39, 0.51], [0.37, 0.55]\rangle$
	P_8	$\langle[0.53, 0.55], [0.18, 0.40]\rangle$	$\langle[0.47, 0.63], [0.58, 0.69]\rangle$	$\langle[0.36, 0.55], [0.39, 0.55]\rangle$

Table 3
Continued

DM_r	Alt.	ζ_9	ζ_{10}
DM_1	P_1	$\langle[0.62, 0.69], [0.39, 0.53]\rangle$	$\langle[0.51, 0.70], [0.28, 0.38]\rangle$
	P_2	$\langle[0.47, 0.54], [0.37, 0.42]\rangle$	$\langle[0.25, 0.27], [0.59, 0.71]\rangle$
	P_3	$\langle[0.57, 0.70], [0.33, 0.52]\rangle$	$\langle[0.44, 0.62], [0.44, 0.50]\rangle$
	P_4	$\langle[0.55, 0.67], [0.47, 0.57]\rangle$	$\langle[0.46, 0.49], [0.39, 0.52]\rangle$
	P_5	$\langle[0.32, 0.37], [0.35, 0.57]\rangle$	$\langle[0.53, 0.66], [0.43, 0.55]\rangle$
	P_6	$\langle[0.59, 0.69], [0.35, 0.50]\rangle$	$\langle[0.63, 0.75], [0.52, 0.63]\rangle$
	P_7	$\langle[0.48, 0.59], [0.53, 0.64]\rangle$	$\langle[0.55, 0.90], [0.55, 0.57]\rangle$
	P_8	$\langle[0.48, 0.56], [0.32, 0.38]\rangle$	$\langle[0.46, 0.55], [0.25, 0.37]\rangle$
DM_2	P_1	$\langle[0.37, 0.54], [0.43, 0.57]\rangle$	$\langle[0.50, 0.58], [0.28, 0.44]\rangle$
	P_2	$\langle[0.48, 0.70], [0.29, 0.43]\rangle$	$\langle[0.50, 0.60], [0.51, 0.72]\rangle$
	P_3	$\langle[0.48, 0.65], [0.32, 0.43]\rangle$	$\langle[0.40, 0.46], [0.42, 0.57]\rangle$
	P_4	$\langle[0.62, 0.66], [0.37, 0.50]\rangle$	$\langle[0.50, 0.59], [0.28, 0.43]\rangle$
	P_5	$\langle[0.37, 0.50], [0.27, 0.38]\rangle$	$\langle[0.53, 0.63], [0.31, 0.46]\rangle$
	P_6	$\langle[0.52, 0.69], [0.32, 0.38]\rangle$	$\langle[0.60, 0.77], [0.52, 0.54]\rangle$
	P_7	$\langle[0.47, 0.60], [0.48, 0.63]\rangle$	$\langle[0.49, 0.67], [0.49, 0.70]\rangle$
	P_8	$\langle[0.54, 0.64], [0.31, 0.40]\rangle$	$\langle[0.43, 0.55], [0.28, 0.40]\rangle$
DM_3	P_1	$\langle[0.57, 0.66], [0.41, 0.52]\rangle$	$\langle[0.39, 0.55], [0.30, 0.44]\rangle$
	P_2	$\langle[0.42, 0.56], [0.29, 0.40]\rangle$	$\langle[0.31, 0.40], [0.53, 0.67]\rangle$
	P_3	$\langle[0.54, 0.79], [0.40, 0.56]\rangle$	$\langle[0.36, 0.58], [0.48, 0.51]\rangle$
	P_4	$\langle[0.53, 0.60], [0.47, 0.53]\rangle$	$\langle[0.54, 0.62], [0.35, 0.52]\rangle$
	P_5	$\langle[0.38, 0.58], [0.34, 0.58]\rangle$	$\langle[0.54, 0.68], [0.43, 0.67]\rangle$
	P_6	$\langle[0.56, 0.64], [0.33, 0.48]\rangle$	$\langle[0.62, 0.71], [0.57, 0.70]\rangle$
	P_7	$\langle[0.48, 0.50], [0.54, 0.60]\rangle$	$\langle[0.48, 0.50], [0.54, 0.77]\rangle$
	P_8	$\langle[0.49, 0.56], [0.31, 0.49]\rangle$	$\langle[0.47, 0.52], [0.26, 0.34]\rangle$

Every recorded entry satisfies $\mathrm{b}^p + \mathrm{d}^q \leq 1$ for $p = 2, q = 3$, the smallest such integer pair admissible across the whole dataset.

Step 1. Construct the UOA and LOA matrices using Eqs. (13)–(14), presented in Table 4.

Table 4
UOA and LOA for the DMs' assessments

	ζ_1	ζ_2	ζ_3	ζ_4
UOA				
P_1	$\langle[0.55, 0.90], [0.28, 0.44]\rangle$	$\langle[0.72, 0.77], [0.25, 0.36]\rangle$	$\langle[0.63, 0.81], [0.46, 0.61]\rangle$	$\langle[0.58, 0.74], [0.39, 0.50]\rangle$
P_2	$\langle[0.57, 0.72], [0.49, 0.62]\rangle$	$\langle[0.58, 0.65], [0.18, 0.27]\rangle$	$\langle[0.75, 0.95], [0.25, 0.36]\rangle$	$\langle[0.51, 0.64], [0.33, 0.40]\rangle$
P_3	$\langle[0.56, 0.73], [0.30, 0.38]\rangle$	$\langle[0.71, 0.84], [0.26, 0.31]\rangle$	$\langle[0.39, 0.52], [0.42, 0.52]\rangle$	$\langle[0.53, 0.68], [0.28, 0.46]\rangle$
P_4	$\langle[0.49, 0.58], [0.23, 0.37]\rangle$	$\langle[0.38, 0.53], [0.37, 0.41]\rangle$	$\langle[0.65, 0.90], [0.41, 0.57]\rangle$	$\langle[0.65, 0.77], [0.43, 0.52]\rangle$
P_5	$\langle[0.60, 0.74], [0.40, 0.50]\rangle$	$\langle[0.69, 0.84], [0.38, 0.51]\rangle$	$\langle[0.59, 0.75], [0.51, 0.64]\rangle$	$\langle[0.73, 0.82], [0.27, 0.45]\rangle$
P_6	$\langle[0.52, 0.67], [0.41, 0.45]\rangle$	$\langle[0.57, 0.73], [0.30, 0.39]\rangle$	$\langle[0.66, 0.79], [0.21, 0.34]\rangle$	$\langle[0.45, 0.63], [0.25, 0.37]\rangle$
P_7	$\langle[0.55, 0.74], [0.25, 0.36]\rangle$	$\langle[0.47, 0.54], [0.36, 0.49]\rangle$	$\langle[0.62, 0.78], [0.30, 0.42]\rangle$	$\langle[0.58, 0.70], [0.24, 0.46]\rangle$
P_8	$\langle[0.44, 0.62], [0.31, 0.44]\rangle$	$\langle[0.44, 0.55], [0.36, 0.44]\rangle$	$\langle[0.58, 0.77], [0.30, 0.40]\rangle$	$\langle[0.48, 0.65], [0.23, 0.29]\rangle$
LOA				
P_1	$\langle[0.40, 0.53], [0.55, 0.57]\rangle$	$\langle[0.53, 0.62], [0.41, 0.54]\rangle$	$\langle[0.56, 0.63], [0.59, 0.68]\rangle$	$\langle[0.57, 0.67], [0.56, 0.65]\rangle$
P_2	$\langle[0.49, 0.65], [0.72, 0.74]\rangle$	$\langle[0.43, 0.54], [0.33, 0.44]\rangle$	$\langle[0.54, 0.67], [0.36, 0.53]\rangle$	$\langle[0.34, 0.49], [0.49, 0.73]\rangle$
P_3	$\langle[0.50, 0.58], [0.51, 0.63]\rangle$	$\langle[0.60, 0.75], [0.29, 0.49]\rangle$	$\langle[0.38, 0.45], [0.47, 0.58]\rangle$	$\langle[0.35, 0.47], [0.34, 0.46]\rangle$
P_4	$\langle[0.36, 0.53], [0.36, 0.46]\rangle$	$\langle[0.30, 0.36], [0.52, 0.66]\rangle$	$\langle[0.55, 0.64], [0.56, 0.75]\rangle$	$\langle[0.55, 0.58], [0.50, 0.63]\rangle$
P_5	$\langle[0.52, 0.64], [0.47, 0.58]\rangle$	$\langle[0.59, 0.67], [0.48, 0.58]\rangle$	$\langle[0.54, 0.64], [0.57, 0.69]\rangle$	$\langle[0.56, 0.70], [0.39, 0.53]\rangle$
P_6	$\langle[0.48, 0.57], [0.53, 0.66]\rangle$	$\langle[0.39, 0.47], [0.44, 0.55]\rangle$	$\langle[0.52, 0.55], [0.42, 0.48]\rangle$	$\langle[0.29, 0.42], [0.30, 0.43]\rangle$
P_7	$\langle[0.48, 0.61], [0.31, 0.47]\rangle$	$\langle[0.39, 0.47], [0.45, 0.60]\rangle$	$\langle[0.53, 0.62], [0.40, 0.59]\rangle$	$\langle[0.41, 0.47], [0.31, 0.49]\rangle$
P_8	$\langle[0.30, 0.49], [0.46, 0.53]\rangle$	$\langle[0.32, 0.45], [0.47, 0.62]\rangle$	$\langle[0.39, 0.43], [0.48, 0.63]\rangle$	$\langle[0.40, 0.45], [0.28, 0.39]\rangle$

Table 4
Continued

	ζ_5	ζ_6	ζ_7
UOA			
P_1	$\langle[0.29, 0.43], [0.32, 0.45]\rangle$	$\langle[0.48, 0.73], [0.21, 0.29]\rangle$	$\langle[0.55, 0.60], [0.53, 0.63]\rangle$
P_2	$\langle[0.42, 0.51], [0.38, 0.52]\rangle$	$\langle[0.52, 0.66], [0.23, 0.30]\rangle$	$\langle[0.46, 0.66], [0.50, 0.60]\rangle$
P_3	$\langle[0.64, 0.90], [0.39, 0.45]\rangle$	$\langle[0.62, 0.71], [0.29, 0.43]\rangle$	$\langle[0.43, 0.59], [0.46, 0.58]\rangle$
P_4	$\langle[0.57, 0.69], [0.43, 0.52]\rangle$	$\langle[0.64, 0.73], [0.37, 0.53]\rangle$	$\langle[0.45, 0.59], [0.45, 0.55]\rangle$
P_5	$\langle[0.70, 0.83], [0.44, 0.54]\rangle$	$\langle[0.50, 0.67], [0.46, 0.50]\rangle$	$\langle[0.46, 0.57], [0.36, 0.51]\rangle$
P_6	$\langle[0.71, 0.77], [0.37, 0.49]\rangle$	$\langle[0.46, 0.61], [0.48, 0.51]\rangle$	$\langle[0.53, 0.70], [0.49, 0.61]\rangle$
P_7	$\langle[0.49, 0.57], [0.42, 0.46]\rangle$	$\langle[0.44, 0.61], [0.48, 0.65]\rangle$	$\langle[0.41, 0.59], [0.39, 0.46]\rangle$
P_8	$\langle[0.57, 0.67], [0.38, 0.53]\rangle$	$\langle[0.53, 0.62], [0.18, 0.28]\rangle$	$\langle[0.47, 0.63], [0.57, 0.68]\rangle$
LOA			
P_1	$\langle[0.28, 0.38], [0.42, 0.55]\rangle$	$\langle[0.44, 0.53], [0.36, 0.54]\rangle$	$\langle[0.42, 0.53], [0.62, 0.72]\rangle$
P_2	$\langle[0.35, 0.46], [0.53, 0.61]\rangle$	$\langle[0.38, 0.47], [0.32, 0.42]\rangle$	$\langle[0.45, 0.57], [0.59, 0.74]\rangle$
P_3	$\langle[0.55, 0.82], [0.55, 0.57]\rangle$	$\langle[0.46, 0.58], [0.40, 0.51]\rangle$	$\langle[0.35, 0.51], [0.55, 0.75]\rangle$
P_4	$\langle[0.41, 0.48], [0.47, 0.58]\rangle$	$\langle[0.54, 0.70], [0.56, 0.77]\rangle$	$\langle[0.37, 0.42], [0.56, 0.71]\rangle$
P_5	$\langle[0.62, 0.73], [0.55, 0.73]\rangle$	$\langle[0.42, 0.52], [0.52, 0.64]\rangle$	$\langle[0.42, 0.52], [0.50, 0.68]\rangle$
P_6	$\langle[0.58, 0.66], [0.51, 0.67]\rangle$	$\langle[0.30, 0.41], [0.55, 0.68]\rangle$	$\langle[0.51, 0.64], [0.57, 0.68]\rangle$
P_7	$\langle[0.34, 0.44], [0.49, 0.59]\rangle$	$\langle[0.44, 0.49], [0.53, 0.72]\rangle$	$\langle[0.34, 0.42], [0.60, 0.70]\rangle$
P_8	$\langle[0.45, 0.65], [0.41, 0.58]\rangle$	$\langle[0.39, 0.53], [0.30, 0.40]\rangle$	$\langle[0.34, 0.45], [0.58, 0.75]\rangle$

Table 4
Continued

	ζ_8	ζ_9	ζ_{10}
UOA			
P_1	$\langle[0.57, 0.70], [0.58, 0.65]\rangle$	$\langle[0.37, 0.54], [0.43, 0.57]\rangle$	$\langle[0.51, 0.70], [0.28, 0.38]\rangle$
P_2	$\langle[0.62, 0.76], [0.59, 0.71]\rangle$	$\langle[0.42, 0.54], [0.37, 0.43]\rangle$	$\langle[0.50, 0.60], [0.51, 0.67]\rangle$
P_3	$\langle[0.37, 0.48], [0.44, 0.60]\rangle$	$\langle[0.48, 0.65], [0.40, 0.56]\rangle$	$\langle[0.44, 0.62], [0.42, 0.50]\rangle$
P_4	$\langle[0.52, 0.60], [0.48, 0.65]\rangle$	$\langle[0.53, 0.60], [0.47, 0.57]\rangle$	$\langle[0.54, 0.62], [0.28, 0.43]\rangle$
P_5	$\langle[0.60, 0.68], [0.37, 0.52]\rangle$	$\langle[0.32, 0.37], [0.35, 0.58]\rangle$	$\langle[0.54, 0.68], [0.31, 0.46]\rangle$
P_6	$\langle[0.53, 0.55], [0.55, 0.57]\rangle$	$\langle[0.52, 0.64], [0.35, 0.50]\rangle$	$\langle[0.63, 0.77], [0.52, 0.54]\rangle$
P_7	$\langle[0.39, 0.51], [0.38, 0.55]\rangle$	$\langle[0.47, 0.50], [0.54, 0.64]\rangle$	$\langle[0.55, 0.90], [0.49, 0.57]\rangle$
P_8	$\langle[0.36, 0.55], [0.46, 0.63]\rangle$	$\langle[0.48, 0.56], [0.32, 0.49]\rangle$	$\langle[0.47, 0.55], [0.25, 0.34]\rangle$
LOA			
P_1	$\langle[0.62, 0.76], [0.47, 0.53]\rangle$	$\langle[0.62, 0.69], [0.39, 0.52]\rangle$	$\langle[0.39, 0.55], [0.30, 0.44]\rangle$
P_2	$\langle[0.68, 0.80], [0.51, 0.66]\rangle$	$\langle[0.48, 0.70], [0.29, 0.40]\rangle$	$\langle[0.25, 0.27], [0.59, 0.72]\rangle$

Table 4
Continued

	ζ_8	ζ_9	ζ_{10}
P_3	$\langle[0.56, 0.62], [0.26, 0.32]\rangle$	$\langle[0.57, 0.79], [0.32, 0.43]\rangle$	$\langle[0.36, 0.46], [0.48, 0.57]\rangle$
P_4	$\langle[0.53, 0.67], [0.35, 0.53]\rangle$	$\langle[0.62, 0.67], [0.37, 0.50]\rangle$	$\langle[0.46, 0.49], [0.39, 0.52]\rangle$
P_5	$\langle[0.60, 0.73], [0.28, 0.42]\rangle$	$\langle[0.38, 0.58], [0.27, 0.38]\rangle$	$\langle[0.53, 0.63], [0.43, 0.67]\rangle$
P_6	$\langle[0.55, 0.90], [0.12, 0.21]\rangle$	$\langle[0.59, 0.69], [0.32, 0.38]\rangle$	$\langle[0.60, 0.71], [0.57, 0.70]\rangle$
P_7	$\langle[0.51, 0.71], [0.36, 0.40]\rangle$	$\langle[0.48, 0.60], [0.48, 0.60]\rangle$	$\langle[0.48, 0.50], [0.55, 0.77]\rangle$
P_8	$\langle[0.55, 0.72], [0.37, 0.39]\rangle$	$\langle[0.54, 0.64], [0.31, 0.38]\rangle$	$\langle[0.43, 0.52], [0.28, 0.40]\rangle$

Step 2. Compute the correlation of each DM's rating profile with the UOA and LOA profiles, per platform, using Eqs. (18)–(19), presented in Table 5.

Table 5
Correlation coefficient values of the DMs from UOA and LOA

	$R(\Gamma_i^r, \Gamma_i^+)$			$R(\Gamma_i^r, \Gamma_i^-)$		
Alt.	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3
P_1	0.3206	−0.1362	−0.1768	−0.0162	−0.0163	0.1066
P_2	0.4803	−0.2948	−0.1960	0.3941	−0.3143	−0.0157
P_3	−0.3124	0.3817	−0.1550	−0.3359	0.3988	−0.1350
P_4	−0.0230	0.1250	−0.0250	0.2068	−0.2628	0.2297
P_5	0.4667	−0.2809	−0.1297	0.3142	−0.2058	−0.0129
P_6	0.2295	−0.0453	−0.1814	−0.1973	−0.1344	0.3045
P_7	0.4246	−0.1909	−0.2759	−0.2963	0.1905	0.1946
P_8	−0.0714	−0.3905	0.4997	0.1744	−0.4290	0.2189
R^{r+}	0.2911	0.2307	0.2049			
R^{r-}				0.2419	0.2440	0.1522

Step 3. Normalize the similarity degrees via Eqs. (21)–(22) to obtain the DM weight vector:

$$\delta = (0.3256,\ 0.3687,\ 0.3057).$$

Step 4. Aggregate the three individual matrices into the collective matrix $\Gamma = (\Gamma_{ij})_{8\times 10}$ via the IVpq-ROFWA operator of Eq. (23), using the weights δ from Step 3, as shown in Tables 6 and 7.

Table 6
Aggregated assessments (ζ_1–ζ_5)

	ζ_1	ζ_2	ζ_3
P_1	$\langle[0.4681, 0.7330], [0.3664, 0.5143]\rangle$	$\langle[0.6041, 0.6898], [0.3332, 0.4546]\rangle$	$\langle[0.5970, 0.7259], [0.5247, 0.6342]\rangle$
P_2	$\langle[0.5294, 0.6804], [0.5741, 0.6921]\rangle$	$\langle[0.5123, 0.5843], [0.2269, 0.3345]\rangle$	$\langle[0.6533, 0.8443], [0.3081, 0.4225]\rangle$
P_3	$\langle[0.5277, 0.6562], [0.4143, 0.5027]\rangle$	$\langle[0.6639, 0.7960], [0.2756, 0.4111]\rangle$	$\langle[0.3868, 0.4823], [0.4424, 0.5381]\rangle$
P_4	$\langle[0.4302, 0.5456], [0.2810, 0.4023]\rangle$	$\langle[0.3489, 0.4511], [0.4172, 0.5209]\rangle$	$\langle[0.5901, 0.8117], [0.5046, 0.6431]\rangle$
P_5	$\langle[0.5550, 0.6924], [0.4288, 0.5322]\rangle$	$\langle[0.6458, 0.7906], [0.4258, 0.5509]\rangle$	$\langle[0.5646, 0.7017], [0.5315, 0.6718]\rangle$
P_6	$\langle[0.5088, 0.6252], [0.4715, 0.5728]\rangle$	$\langle[0.4617, 0.5830], [0.3771, 0.4629]\rangle$	$\langle[0.6195, 0.7255], [0.3136, 0.4152]\rangle$
P_7	$\langle[0.5229, 0.6768], [0.2780, 0.4271]\rangle$	$\langle[0.4377, 0.5077], [0.4210, 0.5517]\rangle$	$\langle[0.5929, 0.7324], [0.3331, 0.4761]\rangle$
P_8	$\langle[0.3995, 0.5791], [0.3796, 0.5030]\rangle$	$\langle[0.4025, 0.5092], [0.4050, 0.5310]\rangle$	$\langle[0.5050, 0.6591], [0.3946, 0.5220]\rangle$

Table 6
Continued

	ζ_4	ζ_5
P_1	$\langle[0.5799, 0.7056], [0.4570, 0.5880]\rangle$	$\langle[0.2866, 0.4010], [0.3667, 0.4956]\rangle$
P_2	$\langle[0.4203, 0.5516], [0.3920, 0.5313]\rangle$	$\langle[0.3767, 0.4815], [0.4637, 0.5525]\rangle$
P_3	$\langle[0.4168, 0.5671], [0.3137, 0.4590]\rangle$	$\langle[0.5915, 0.8571], [0.4570, 0.4980]\rangle$
P_4	$\langle[0.5996, 0.7109], [0.4569, 0.5686]\rangle$	$\langle[0.4873, 0.5956], [0.4416, 0.5477]\rangle$

Table 6
Continued

	ζ_4	ζ_5
P_5	$\langle[0.6411, 0.7815], [0.3216, 0.4932]\rangle$	$\langle[0.6670, 0.7981], [0.5115, 0.6372]\rangle$
P_6	$\langle[0.3895, 0.5231], [0.2720, 0.3896]\rangle$	$\langle[0.6374, 0.7286], [0.4260, 0.5506]\rangle$
P_7	$\langle[0.4928, 0.6187], [0.2834, 0.4720]\rangle$	$\langle[0.4175, 0.5094], [0.4514, 0.5406]\rangle$
P_8	$\langle[0.4538, 0.5803], [0.2464, 0.3417]\rangle$	$\langle[0.5289, 0.6623], [0.3937, 0.5567]\rangle$

Table 7
Aggregated assessments (ζ_6–ζ_{10})

	ζ_6	ζ_7	ζ_8
P_1	$\langle[0.4558, 0.6360], [0.3025, 0.4166]\rangle$	$\langle[0.4785, 0.5717], [0.5653, 0.6646]\rangle$	$\langle[0.5973, 0.7308], [0.5118, 0.6113]\rangle$
P_2	$\langle[0.4394, 0.5640], [0.2686, 0.3690]\rangle$	$\langle[0.4525, 0.6156], [0.5320, 0.6632]\rangle$	$\langle[0.6437, 0.7779], [0.5371, 0.6768]\rangle$
P_3	$\langle[0.5437, 0.6439], [0.3442, 0.4683]\rangle$	$\langle[0.3998, 0.5428], [0.5157, 0.6538]\rangle$	$\langle[0.4583, 0.5643], [0.3666, 0.4603]\rangle$
P_4	$\langle[0.5947, 0.7146], [0.4552, 0.5925]\rangle$	$\langle[0.4072, 0.5071], [0.4979, 0.6274]\rangle$	$\langle[0.5254, 0.6406], [0.4075, 0.5806]\rangle$
P_5	$\langle[0.4751, 0.5955], [0.4878, 0.5815]\rangle$	$\langle[0.4414, 0.5427], [0.4421, 0.5986]\rangle$	$\langle[0.6000, 0.7048], [0.3289, 0.4602]\rangle$
P_6	$\langle[0.3745, 0.4973], [0.5225, 0.6128]\rangle$	$\langle[0.5199, 0.6629], [0.5350, 0.6346]\rangle$	$\langle[0.5428, 0.7372], [0.2480, 0.3678]\rangle$
P_7	$\langle[0.4436, 0.5740], [0.5078, 0.6795]\rangle$	$\langle[0.3804, 0.5307], [0.4701, 0.5773]\rangle$	$\langle[0.4651, 0.5995], [0.3710, 0.4959]\rangle$
P_8	$\langle[0.4438, 0.5658], [0.2303, 0.3444]\rangle$	$\langle[0.4080, 0.5468], [0.5709, 0.7055]\rangle$	$\langle[0.4642, 0.6281], [0.4069, 0.5180]\rangle$

Table 7
Continued

	ζ_9	ζ_{10}
P_1	$\langle[0.5331, 0.6300], [0.4072, 0.5415]\rangle$	$\langle[0.4753, 0.6196], [0.2867, 0.4162]\rangle$
P_2	$\langle[0.4597, 0.6165], [0.3125, 0.4164]\rangle$	$\langle[0.3793, 0.4626], [0.5379, 0.7022]\rangle$
P_3	$\langle[0.5259, 0.7172], [0.3428, 0.4972]\rangle$	$\langle[0.4023, 0.5582], [0.4470, 0.5275]\rangle$
P_4	$\langle[0.5700, 0.6481], [0.4308, 0.5311]\rangle$	$\langle[0.4979, 0.5758], [0.3323, 0.4862]\rangle$
P_5	$\langle[0.3584, 0.4904], [0.3135, 0.4948]\rangle$	$\langle[0.5347, 0.6562], [0.3803, 0.5469]\rangle$
P_6	$\langle[0.5525, 0.6775], [0.3314, 0.4462]\rangle$	$\langle[0.6176, 0.7484], [0.5378, 0.6161]\rangle$
P_7	$\langle[0.4777, 0.5666], [0.5116, 0.6231]\rangle$	$\langle[0.5096, 0.7557], [0.5224, 0.6724]\rangle$
P_8	$\langle[0.5040, 0.5883], [0.3145, 0.4179]\rangle$	$\langle[0.4539, 0.5422], [0.2640, 0.3693]\rangle$

Step 5. Normalize the aggregated matrix via Eq. (24), swapping the membership and non-membership intervals for the cost-type attributes ζ_8 and ζ_9, to obtain $\widetilde{\Gamma} = (\widetilde{\Gamma}_{ij})_{8\times 10}$, presented in Tables 8 and 9.

Table 8
Normalized assessments (ζ_1–ζ_5)

	ζ_1	ζ_2	ζ_3
P_1	$\langle[0.4681, 0.7330], [0.3664, 0.5143]\rangle$	$\langle[0.6041, 0.6898], [0.3332, 0.4546]\rangle$	$\langle[0.5970, 0.7259], [0.5247, 0.6342]\rangle$
P_2	$\langle[0.5294, 0.6804], [0.5741, 0.6921]\rangle$	$\langle[0.5123, 0.5843], [0.2269, 0.3345]\rangle$	$\langle[0.6533, 0.8443], [0.3081, 0.4225]\rangle$
P_3	$\langle[0.5277, 0.6562], [0.4143, 0.5027]\rangle$	$\langle[0.6639, 0.7960], [0.2756, 0.4111]\rangle$	$\langle[0.3868, 0.4823], [0.4424, 0.5381]\rangle$
P_4	$\langle[0.4302, 0.5456], [0.2810, 0.4023]\rangle$	$\langle[0.3489, 0.4511], [0.4172, 0.5209]\rangle$	$\langle[0.5901, 0.8117], [0.5046, 0.6431]\rangle$
P_5	$\langle[0.5550, 0.6924], [0.4288, 0.5322]\rangle$	$\langle[0.6458, 0.7906], [0.4258, 0.5509]\rangle$	$\langle[0.5646, 0.7017], [0.5315, 0.6718]\rangle$
P_6	$\langle[0.5088, 0.6252], [0.4715, 0.5728]\rangle$	$\langle[0.4617, 0.5830], [0.3771, 0.4629]\rangle$	$\langle[0.6195, 0.7255], [0.3136, 0.4152]\rangle$
P_7	$\langle[0.5229, 0.6768], [0.2780, 0.4271]\rangle$	$\langle[0.4377, 0.5077], [0.4210, 0.5517]\rangle$	$\langle[0.5929, 0.7324], [0.3331, 0.4761]\rangle$
P_8	$\langle[0.3995, 0.5791], [0.3796, 0.5030]\rangle$	$\langle[0.4025, 0.5092], [0.4050, 0.5310]\rangle$	$\langle[0.5050, 0.6591], [0.3946, 0.5220]\rangle$

Table 8
Continued

	ζ_4	ζ_5
P_1	$\langle[0.5799, 0.7056], [0.4570, 0.5880]\rangle$	$\langle[0.2866, 0.4010], [0.3667, 0.4956]\rangle$
P_2	$\langle[0.4203, 0.5516], [0.3920, 0.5313]\rangle$	$\langle[0.3767, 0.4815], [0.4637, 0.5525]\rangle$
P_3	$\langle[0.4168, 0.5671], [0.3137, 0.4590]\rangle$	$\langle[0.5915, 0.8571], [0.4570, 0.4980]\rangle$

Table 8
Continued

	ζ_4	ζ_5
P_4	$\langle[0.5996, 0.7109], [0.4569, 0.5686]\rangle$	$\langle[0.4873, 0.5956], [0.4416, 0.5477]\rangle$
P_5	$\langle[0.6411, 0.7815], [0.3216, 0.4932]\rangle$	$\langle[0.6670, 0.7981], [0.5115, 0.6372]\rangle$
P_6	$\langle[0.3895, 0.5231], [0.2720, 0.3896]\rangle$	$\langle[0.6374, 0.7286], [0.4260, 0.5506]\rangle$
P_7	$\langle[0.4928, 0.6187], [0.2834, 0.4720]\rangle$	$\langle[0.4175, 0.5094], [0.4514, 0.5406]\rangle$
P_8	$\langle[0.4538, 0.5803], [0.2464, 0.3417]\rangle$	$\langle[0.5289, 0.6623], [0.3937, 0.5567]\rangle$

Table 9
Normalized assessments (ζ_6–ζ_{10}; note ζ_8, ζ_9 swapped relative to Table 7)

	ζ_6	ζ_7	ζ_8
P_1	$\langle[0.4558, 0.6360], [0.3025, 0.4166]\rangle$	$\langle[0.4785, 0.5717], [0.5653, 0.6646]\rangle$	$\langle[0.5118, 0.6113], [0.5973, 0.7308]\rangle$
P_2	$\langle[0.4394, 0.5640], [0.2686, 0.3690]\rangle$	$\langle[0.4525, 0.6156], [0.5320, 0.6632]\rangle$	$\langle[0.5371, 0.6768], [0.6437, 0.7779]\rangle$
P_3	$\langle[0.5437, 0.6439], [0.3442, 0.4683]\rangle$	$\langle[0.3998, 0.5428], [0.5157, 0.6538]\rangle$	$\langle[0.3666, 0.4603], [0.4583, 0.5643]\rangle$
P_4	$\langle[0.5947, 0.7146], [0.4552, 0.5925]\rangle$	$\langle[0.4072, 0.5071], [0.4979, 0.6274]\rangle$	$\langle[0.4075, 0.5806], [0.5254, 0.6406]\rangle$
P_5	$\langle[0.4751, 0.5955], [0.4878, 0.5815]\rangle$	$\langle[0.4414, 0.5427], [0.4421, 0.5986]\rangle$	$\langle[0.3289, 0.4602], [0.6000, 0.7048]\rangle$
P_6	$\langle[0.3745, 0.4973], [0.5225, 0.6128]\rangle$	$\langle[0.5199, 0.6629], [0.5350, 0.6346]\rangle$	$\langle[0.2480, 0.3678], [0.5428, 0.7372]\rangle$
P_7	$\langle[0.4436, 0.5740], [0.5078, 0.6795]\rangle$	$\langle[0.3804, 0.5307], [0.4701, 0.5773]\rangle$	$\langle[0.3710, 0.4959], [0.4651, 0.5995]\rangle$
P_8	$\langle[0.4438, 0.5658], [0.2303, 0.3444]\rangle$	$\langle[0.4080, 0.5468], [0.5709, 0.7055]\rangle$	$\langle[0.4069, 0.5180], [0.4642, 0.6281]\rangle$

Table 9
Continued

	ζ_9	ζ_{10}
P_1	$\langle[0.4072, 0.5415], [0.5331, 0.6300]\rangle$	$\langle[0.4753, 0.6196], [0.2867, 0.4162]\rangle$
P_2	$\langle[0.3125, 0.4164], [0.4597, 0.6165]\rangle$	$\langle[0.3793, 0.4626], [0.5379, 0.7022]\rangle$
P_3	$\langle[0.3428, 0.4972], [0.5259, 0.7172]\rangle$	$\langle[0.4023, 0.5582], [0.4470, 0.5275]\rangle$
P_4	$\langle[0.4308, 0.5311], [0.5700, 0.6481]\rangle$	$\langle[0.4979, 0.5758], [0.3323, 0.4862]\rangle$
P_5	$\langle[0.3135, 0.4948], [0.3584, 0.4904]\rangle$	$\langle[0.5347, 0.6562], [0.3803, 0.5469]\rangle$
P_6	$\langle[0.3314, 0.4462], [0.5525, 0.6775]\rangle$	$\langle[0.6176, 0.7484], [0.5378, 0.6161]\rangle$
P_7	$\langle[0.5116, 0.6231], [0.4777, 0.5666]\rangle$	$\langle[0.5096, 0.7557], [0.5224, 0.6724]\rangle$
P_8	$\langle[0.3145, 0.4179], [0.5040, 0.5883]\rangle$	$\langle[0.4539, 0.5422], [0.2640, 0.3693]\rangle$

Step 6. Fix the fuzzy acceptability threshold $\Gamma^0 = \langle[0.30, 0.45], [0.30, 0.40]\rangle$. Using the score function of Definition 3 with $p = 2, q = 3$, this threshold scores $S(\Gamma^0) = 0.4111$.
Step 7. Solve the LP model of Eqs. (39)–(40) independently for each of the eight platforms. The computed scores are

$$(-2.9875,\ -3.8433,\ -3.7626,\ -2.7379,\ -3.3141,\ -3.2698,\ -3.0629,\ -3.1396).$$

Each solution was reproduced with a second linear-programming algorithm and cross-checked against an exhaustive search over every single-attribute weight allocation, both returning identical values.
Step 8. Rank the platforms in ascending order of their scores, a lower score denoting a more preferable platform. The final ranking is

$$P_2 \succ P_3 \succ P_5 \succ P_6 \succ P_8 \succ P_7 \succ P_1 \succ P_4.$$

Platform P_2 is therefore identified as the most suitable telehealth platform for rural deployment under the proposed IVpq-ROF decision-making framework.

5.2 Validation

Building on the notation and framework established thus far, validation of the LP-based ranking is carried out through an integrated CRITIC–Exp-TODIM procedure adapted to the IVpq-ROF setting [29].

5.2.1 Attribute weighting via CRITIC

The construction of the individual decision matrices, the benefit/cost normalization, and their aggregation into the collective matrix $\Gamma = (\Gamma_{ij})_{n\times m}$ have already been carried out in Steps 1–5 of Section 5.1; the normalized matrix $\widetilde{\Gamma} = (\widetilde{\Gamma}_{ij})_{n\times m}$ obtained there (Tables 8–9) is taken as the starting point below, so that no part of the aggregation pipeline is repeated.

Step 1. For every entry $\widetilde{\Gamma}_{ij} = \langle[\widetilde{\mathsf{a}}_{ij}, \widetilde{\mathsf{b}}_{ij}], [\widetilde{\mathsf{c}}_{ij}, \widetilde{\mathsf{d}}_{ij}]\rangle$, a score value is computed through

$$\widehat{S}(\widetilde{\Gamma}_{ij}) = \frac{1}{2}\left[\begin{array}{c} \left(\widetilde{\mathsf{a}}^p_{ij} - \widetilde{\mathsf{c}}^q_{ij}\right)\left(1 + \sqrt[\ell]{1 - \left(\widetilde{\mathsf{a}}^p_{ij} + \widetilde{\mathsf{c}}^q_{ij}\right)}\right) \\ +\left(\widetilde{\mathsf{b}}^p_{ij} - \widetilde{\mathsf{d}}^q_{ij}\right)\left(1 + \sqrt[\ell]{1 - \left(\widetilde{\mathsf{b}}^p_{ij} + \widetilde{\mathsf{d}}^q_{ij}\right)}\right) \end{array}\right], \tag{41}$$

with $\ell = \mathrm{lcm}(p, q)$ as in Definition 1. This score, distinct from the ranking-oriented score of Definition 3, serves solely as an intermediate device for extracting the attribute weights below. Table 10 reports $\widehat{S}(\widetilde{\Gamma}_{ij})$ for every platform–attribute pair.

Table 10
Score matrix $\widehat{S}(\widetilde{\Gamma}_{ij})$

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}
P_1	0.5328	0.6714	0.4418	0.4977	0.0704	0.4913	0.0771	0.0315	0.0544	0.4943
P_2	0.2027	0.5367	0.9431	0.2607	0.1018	0.4283	0.1321	0.0100	−0.0579	−0.1352
P_3	0.4844	0.9116	0.1354	0.3553	0.7687	0.5405	0.0362	0.0682	−0.1400	0.2276
P_4	0.3840	0.1082	0.5324	0.5445	0.3262	0.5209	0.0508	0.0893	0.0095	0.4120
P_5	0.5238	0.7267	0.3273	0.8027	0.6092	0.2541	0.1812	−0.2339	0.1738	0.4715
P_6	0.3378	0.3852	0.7605	0.3371	0.6428	0.0140	0.2794	−0.3458	−0.1629	0.5006
P_7	0.6022	0.1999	0.6968	0.4776	0.1774	0.0781	0.1246	0.0652	0.3398	0.3404
P_8	0.3007	0.1985	0.4603	0.4727	0.4581	0.4505	−0.0669	0.0832	−0.0564	0.4191

Step 2. The score matrix is rescaled column-wise onto $[0, 1]$,

$$\ddot{S}(\widetilde{\Gamma}_{ij}) = \frac{\widehat{S}(\widetilde{\Gamma}_{ij}) - \min_i \widehat{S}(\widetilde{\Gamma}_{ij})}{\max_i \widehat{S}(\widetilde{\Gamma}_{ij}) - \min_i \widehat{S}(\widetilde{\Gamma}_{ij})}, \tag{42}$$

giving the normalized score matrix of Table 11.

Table 11
Normalized score matrix $\ddot{S}(\widetilde{\Gamma}_{ij})$

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}
P_1	0.8264	0.7010	0.3794	0.4373	0.0000	0.9067	0.4160	0.8672	0.4324	0.9901
P_2	0.0000	0.5333	1.0000	0.0000	0.0449	0.7869	0.5748	0.8177	0.2089	0.0000
P_3	0.7051	1.0000	0.0000	0.1746	1.0000	1.0000	0.2979	0.9515	0.0456	0.5706
P_4	0.4538	0.0000	0.4915	0.5237	0.3663	0.9628	0.3400	1.0000	0.3431	0.8606
P_5	0.8037	0.7699	0.2376	1.0000	0.7716	0.4560	0.7164	0.2572	0.6699	0.9542
P_6	0.3382	0.3447	0.7739	0.1410	0.8198	0.0000	1.0000	0.0000	0.0000	1.0000
P_7	1.0000	0.1141	0.6950	0.4002	0.1532	0.1218	0.5532	0.9445	1.0000	0.7480
P_8	0.2454	0.1124	0.4023	0.3912	0.5552	0.8291	0.0000	0.9859	0.2120	0.8717

Step 3. Treating the eight column entries of Table 11 as ordinary observations, the correlation coefficient between attributes ζ_j and ζ_r is computed as

$$\zeta_{jr} = \frac{\sum_{i=1}^{n}\left(\ddot{S}(\widetilde{\Gamma}_{ij}) - \overline{\ddot{S}}(\zeta_j)\right)\left(\ddot{S}(\widetilde{\Gamma}_{ir}) - \overline{\ddot{S}}(\zeta_r)\right)}{\sqrt{\sum_{i=1}^{n}\left(\ddot{S}(\widetilde{\Gamma}_{ij}) - \overline{\ddot{S}}(\zeta_j)\right)^2 \sum_{i=1}^{n}\left(\ddot{S}(\widetilde{\Gamma}_{ir}) - \overline{\ddot{S}}(\zeta_r)\right)^2}}, \tag{43}$$

where $\overline{\ddot{S}}(\zeta_j) = \frac{1}{n}\sum_{i=1}^{n}\ddot{S}(\widetilde{\Gamma}_{ij})$. The resulting 10×10 correlation matrix is given in Table 12.

Step 4. The standard deviation of each attribute's normalized score column, reflecting its discriminating power across the eight platforms, follows from

$$\sigma_j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\ddot{S}(\widetilde{\Gamma}_{ij}) - \overline{\ddot{S}}(\zeta_j)\right)^2}, \qquad j = 1, \ldots, m. \tag{44}$$

Step 5. The information carried by attribute ζ_j, combining its own contrast with its conflict against

Table 12
Correlation matrix (ζ_{jr}) between attributes

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}
ζ_1	1.000	0.240	−0.517	0.538	0.033	−0.197	0.062	0.056	0.680	0.502
ζ_2	0.240	1.000	−0.504	0.023	0.334	0.253	0.149	−0.198	−0.222	−0.180
ζ_3	−0.517	−0.504	1.000	−0.472	−0.560	−0.446	0.434	−0.171	0.044	−0.420
ζ_4	0.538	0.023	−0.472	1.000	0.134	−0.045	−0.007	−0.176	0.577	0.587
ζ_5	0.033	0.334	−0.560	0.134	1.000	−0.124	0.122	−0.450	−0.429	0.291
ζ_6	−0.197	0.253	−0.446	−0.045	−0.124	1.000	−0.753	0.677	−0.335	−0.271
ζ_7	0.062	0.149	0.434	−0.007	0.122	−0.753	1.000	−0.848	0.063	0.072
ζ_8	0.056	−0.198	−0.171	−0.176	−0.450	0.677	−0.848	1.000	0.155	−0.326
ζ_9	0.680	−0.222	0.044	0.577	−0.429	−0.335	0.063	0.155	1.000	0.178
ζ_{10}	0.502	−0.180	−0.420	0.587	0.291	−0.271	0.072	−0.326	0.178	1.000

every other attribute, is

$$\mathcal{I}_j = \sigma_j \sum_{r=1}^{m} \big(1 - \zeta_{jr}\big), \qquad j = 1, \ldots, m. \tag{45}$$

Step 6. Normalizing $\mathcal{I}_j$ across all ten attributes yields the CRITIC weight vector,

$$\omega_j = \frac{\mathcal{I}_j}{\sum_{j=1}^{m} \mathcal{I}_j}, \qquad j = 1, \ldots, m. \tag{46}$$

Table 13 collects σ_j, $\mathcal{I}_j$ and ω_j for the ten attributes.

Table 13
Standard deviations, information content, and CRITIC weights

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6	ζ_7	ζ_8	ζ_9	ζ_{10}
σ_j	0.3416	0.3616	0.3176	0.3053	0.3806	0.3917	0.2994	0.3810	0.3347	0.3346
$\mathcal{I}_j$	2.5969	3.2915	3.6878	2.3937	3.6721	4.0112	2.9056	3.9173	2.7744	2.8666
ω_j	0.0809	0.1025	0.1148	0.0745	0.1143	0.1249	0.0905	0.1220	0.0864	0.0893

The resulting weight vector, $\omega = (0.0809, 0.1025, 0.1148, 0.0745, 0.1143, 0.1249, 0.0905, 0.1220, 0.0864, 0.0893)$, satisfies $\sum_{j=1}^{m} \omega_j = 1$ and is carried forward into the ranking stage below.

5.2.2 Ranking via Exponential TODIM

Steps 1–6 above already supply the decision matrix, its normalization, its aggregation, and the CRITIC weight vector ω; the exponential TODIM (Exp-TODIM) procedure adopted here re-uses all four without repetition and proceeds directly to the relative-weight and dominance stages.

Step 7. The attribute weights are rescaled relative to the most informative attribute,

$$\varpi_j = \frac{\omega_j}{\omega_t}, \qquad j = 1, \ldots, m, \qquad \omega_t = \max_{1 \le j \le m} \{\omega_j\}, \tag{47}$$

which, since $\omega_6 = 0.1249$ is the largest component, identifies ζ_6 (resource efficiency under low-bandwidth conditions) as the reference attribute and yields

$$\varpi = (0.6474, 0.8206, 0.9194, 0.5968, 0.9155, 1.0000, 0.7244, 0.9766, 0.6917, 0.7146).$$

Step 8. Comparing platforms attribute by attribute requires a distance measure between two IVpq-ROFNs, defined next.

Definition 10. *[29] Let $\Gamma_1 = \langle [\mathsf{a}_1, \mathsf{b}_1], [\mathsf{c}_1, \mathsf{d}_1] \rangle$ and $\Gamma_2 = \langle [\mathsf{a}_2, \mathsf{b}_2], [\mathsf{c}_2, \mathsf{d}_2] \rangle$ be any two IVpq-ROFNs, with hesitancy bounds $\pi_1^\ell = 1 - \mathsf{a}_1^p - \mathsf{c}_1^q$, $\bar{\pi}_1^\ell = 1 - \mathsf{b}_1^p - \mathsf{d}_1^q$ (and similarly $\pi_2^\ell, \bar{\pi}_2^\ell$ for Γ_2) obtained by raising the hesitancy degree of Definition 1 to the power $\ell = \mathrm{lcm}(p, q)$. The distance between Γ_1 and Γ_2 is*

$$\mathfrak{d}(\Gamma_1, \Gamma_2) = \frac{1}{4} \left(\left|\mathsf{a}_1^p - \mathsf{a}_2^p\right| + \left|\mathsf{b}_1^p - \mathsf{b}_2^p\right| + \left|\mathsf{c}_1^q - \mathsf{c}_2^q\right| + \left|\mathsf{d}_1^q - \mathsf{d}_2^q\right| + \left|\pi_1^\ell - \pi_2^\ell\right| + \left|\bar{\pi}_1^\ell - \bar{\pi}_2^\ell\right| \right). \tag{48}$$

Applied to a pair of platforms P_i, P_k on attribute ζ_j, this gives $\mathfrak{d}\big(\widetilde{\Gamma}_{ij}, \widetilde{\Gamma}_{kj}\big)$.

Step 9. For platforms P_i and P_k ($i \neq k$), the dominance of P_i over P_k on attribute ζ_j is

$$\Phi_j(P_i, P_k) = \begin{cases} \dfrac{\varpi_j\Big(1 - 10^{-\rho\,\mathfrak{d}(\widetilde{\Gamma}_{ij},\widetilde{\Gamma}_{kj})}\Big)}{\sum\limits_{j=1}^{m}\varpi_j}, & \widehat{S}(\widetilde{\Gamma}_{ij}) > \widehat{S}(\widetilde{\Gamma}_{kj}), \\ 0, & \widehat{S}(\widetilde{\Gamma}_{ij}) = \widehat{S}(\widetilde{\Gamma}_{kj}), \\ -\dfrac{1}{\delta}\dfrac{\sum\limits_{r=1}^{m}\varpi_r\Big(1 - 10^{-\rho\,\mathfrak{d}(\widetilde{\Gamma}_{ir},\widetilde{\Gamma}_{kr})}\Big)}{\varpi_j}, & \widehat{S}(\widetilde{\Gamma}_{ij}) < \widehat{S}(\widetilde{\Gamma}_{kj}), \end{cases} \tag{49}$$

where $\rho > 0$ governs how quickly the perceived gain saturates with distance and $\delta > 0$ is the loss-aversion coefficient amplifying the impact of a loss relative to an equally sized gain. As the source description leaves ρ, δ as free parameters without prescribing numerical values, we adopt $\rho = 1$ and the widely used prospect-theory value $\delta = 2.25$; a sensitivity check over $\rho \in \{0.5, 1, 2\}$ and $\delta \in \{1, 2.25\}$ left the top five platforms unchanged, so the ranking is not an artefact of this choice.

Step 10. Summing the per-attribute preference indices over all attributes and all competing platforms gives the total dominance degree of P_i,

$$\mathcal{D}_i = \sum_{k=1}^{n}\sum_{j=1}^{m}\Phi_j(P_i, P_k), \qquad i = 1, \ldots, n. \tag{50}$$

Step 11. Rescaling $\mathcal{D}_i$ onto $[0, 1]$ gives the overall dominance value of each platform,

$$F(P_i) = \frac{\mathcal{D}_i - \min_i \mathcal{D}_i}{\max_i \mathcal{D}_i - \min_i \mathcal{D}_i}, \qquad i = 1, \ldots, n, \tag{51}$$

with a larger $F(P_i)$ denoting a more preferable platform. Table 14 reports $\mathcal{D}_i$ and $F(P_i)$ for all eight platforms.

Table 14
Dominance degrees and overall dominance values

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
$\mathcal{D}_i$	−38.5058	−71.4670	−52.3886	−44.1387	−31.8056	−52.4922	−39.3787	−54.4773
$F(P_i)$	0.8311	0.0000	0.4810	0.6890	1.0000	0.4784	0.8091	0.4284

Step 12. Ranking the platforms in descending order of $F(P_i)$ gives

$$P_5 \succ P_1 \succ P_7 \succ P_4 \succ P_3 \succ P_6 \succ P_8 \succ P_2.$$

The CRITIC–Exp-TODIM ranking agrees with the LP-based ranking of Section 5.1 in placing P_2 firmly at the bottom of the order and in keeping P_4 in the lower-middle tier under both methods, but it favours P_5 and P_1 for the leading positions, whereas the LP model favoured P_2 and P_3. This divergence is not unexpected: the LP model scores each platform under its own most favourable, endogenously derived attribute weighting in a DEA-like fashion, whereas Exp-TODIM applies one shared, CRITIC-derived weighting to every platform and further reshapes the comparison through a loss-averse, exponential gain/loss function operating on the full membership–non-membership–hesitancy profile of Definition 10. The two methods therefore encode different notions of ”best” (best attainable potential versus best pairwise-dominant performance under uniform weights and psychological loss aversion), and a degree of disagreement between them is itself informative: it signals that the platforms' relative standing is sensitive to whether attribute importance is treated as alternative-specific and self-optimized or as fixed and shared, a distinction worth flagging to decision-makers rather than concealing behind an artificially forced consensus.

5.3 Rank Fusion using Copeland Approach

The individual DM-level rankings produced by the LP model are consolidated here into a single consensus ordering, using the previously derived DM weights δ, so that the group-level ranking of Section 5.1 can be cross-checked against a ranking built entirely from individual judgements.

Step 1. The LP model of Eqs. (39)–(40) is re-solved separately for each DM's own (unaggregated) normalized assessment matrix, giving a rank RM_{ir} for platform P_i under DM_r; the resulting 8×3 rank matrix is shown in Table 15.

Table 15
Ranks from individual DMs' LP-based assessments

	DM_1	DM_2	DM_3
P_1	2	8	7
P_2	1	4	1
P_3	5	2	2
P_4	8	5	8
P_5	4	6	3
P_6	7	1	6
P_7	6	3	4
P_8	3	7	5

Step 2. The weighted rank matrix follows from

$$WRM_{ir} = \delta^r \cdot RM_{ir}, \qquad i = 1, \ldots, n, \quad r = 1, \ldots, l, \tag{52}$$

using the DM weights $\delta = (0.3256, 0.3687, 0.3057)$ obtained in Step 3 of Section 5.1.

Step 3. Summing across DMs gives the total weighted score of each platform,

$$M_i = \sum_{r=1}^{l} WRM_{ir}, \qquad i = 1, \ldots, n, \tag{53}$$

and its deviation from the best-performing platform is

$$N_i = M_{\text{max}} - M_i, \qquad M_{\text{max}} = \max_{1 \le i \le n} M_i. \tag{54}$$

Step 4. The net ordering value combining both quantities is

$$O_i = M_i - N_i, \qquad i = 1, \ldots, n, \tag{55}$$

normalized to

$$\bar{O}_i = \frac{O_i}{\sum_{i=1}^{n} O_i}, \qquad i = 1, \ldots, n, \tag{56}$$

with a smaller $\bar{O}_i$ marking a more preferred platform. Table 16 reports WRM_{ir}, M_i, N_i, O_i, and $\bar{O}_i$ for every platform.

Table 16
Copeland fusion results

	WRM_{ir}							
	DM_1	DM_2	DM_3	M_i	N_i	O_i	$\bar{O}_i$	Fused rank
P_1	0.6512	2.9496	2.1399	5.7407	1.1532	4.5875	0.2723	7
P_2	0.3256	1.4748	0.3057	2.1061	4.7878	−2.6817	−0.1592	1
P_3	1.6280	0.7374	0.6114	2.9768	3.9171	−0.9403	−0.0558	2
P_4	2.6048	1.8435	2.4456	6.8939	0.0000	6.8939	0.4092	8
P_5	1.3024	2.2122	0.9171	4.4317	2.4622	1.9695	0.1169	4
P_6	2.2792	0.3687	1.8342	4.4821	2.4118	2.0703	0.1229	5
P_7	1.9536	1.1061	1.2228	4.2825	2.6114	1.6711	0.0992	3
P_8	0.9768	2.5809	1.5285	5.0862	1.8077	3.2785	0.1946	6

Step 5. Ranking the platforms in ascending order of $\bar{O}_i$ gives the fused ranking

$$P_2 \succ P_3 \succ P_7 \succ P_5 \succ P_6 \succ P_8 \succ P_1 \succ P_4.$$

Figure 2 contrasts the individual DM rankings of Table 15 with this fused ordering. The Copeland-fused ranking places P_2 and P_3 at the top and P_4 at the bottom, exactly as the group-level LP ranking of Section 5.1 does; the two orderings differ only in how P_5, P_6, P_7, P_8 are interleaved in the middle tier, a pattern consistent with the individual DMs' visibly divergent rankings in Table 15 and with the general expectation that a rank-based consensus need not reproduce every mid-table position of a score-based aggregate.

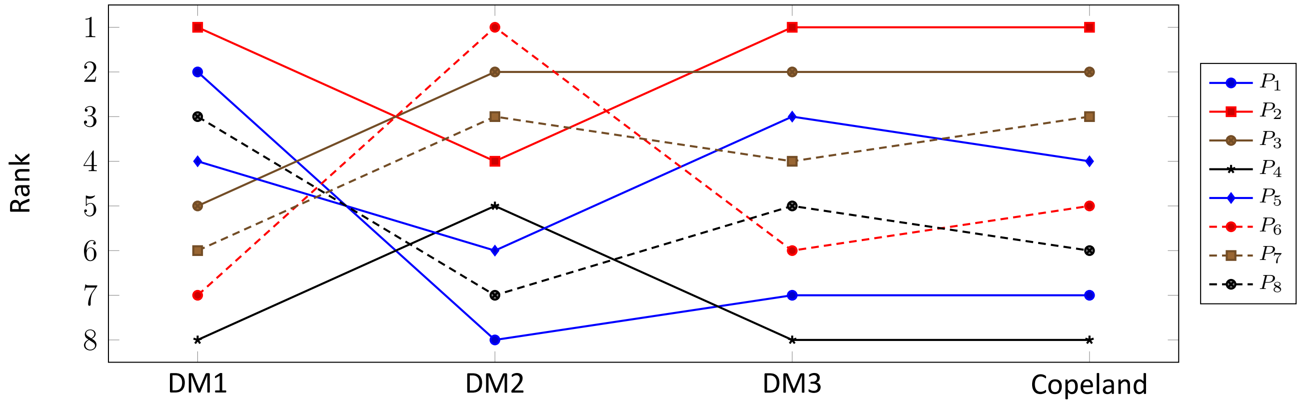

Fig. 2. Ranking from individual DMs and the Copeland-fused consensus

6. Sensitivity Analysis

Since the proposed framework carries two rung parameters rather than one, its stability is examined along both directions independently: first holding the non-membership rung q fixed at its minimal admissible value while p is varied, then holding the membership rung p fixed while q is varied. In both cases the entire pipeline, correlation-based DM weighting, IVpq-ROFWA aggregation, and the per-alternative LP, is re-executed at each parameter setting, and the resulting score vector is compared with the base case $(p, q) = (2, 3)$ through its standard deviation of score differences and its Pearson correlation with the base-case scores.

Table 17
Sensitivity analysis for q (with $p = 2$ fixed)

q	Scores	Ranking	Std. Dev.	Correlation
3	$(-2.9875, -3.8433, -3.7626, -2.7379, -3.3141, -3.2698, -3.0629, -3.1396)$	$P_2 \succ P_3 \succ P_5 \succ P_6 \succ P_8 \succ P_7 \succ P_1 \succ P_4$	0.0000	1.0000
4	$(-2.9926, -3.8246, -3.7640, -2.7468, -3.3240, -3.2681, -3.0665, -3.1454)$	$P_2 \succ P_3 \succ P_5 \succ P_6 \succ P_8 \succ P_7 \succ P_1 \succ P_4$	0.0085	0.9999
5	$(-2.9945, -3.8157, -3.7649, -2.7503, -3.3278, -3.2684, -3.0679, -3.1474)$	$P_2 \succ P_3 \succ P_5 \succ P_6 \succ P_8 \succ P_7 \succ P_1 \succ P_4$	0.0122	0.9997
6	$(-2.9942, -3.8118, -3.7660, -2.7502, -3.3279, -3.2708, -3.0679, -3.1469)$	$P_2 \succ P_3 \succ P_5 \succ P_6 \succ P_8 \succ P_7 \succ P_1 \succ P_4$	0.0134	0.9996
7	$(-2.9929, -3.8103, -3.7671, -2.7486, -3.3263, -3.2740, -3.0672, -3.1453)$	$P_2 \succ P_3 \succ P_5 \succ P_6 \succ P_8 \succ P_7 \succ P_1 \succ P_4$	0.0134	0.9996
8	$(-2.9914, -3.8101, -3.7679, -2.7466, -3.3243, -3.2769, -3.0664, -3.1435)$	$P_2 \succ P_3 \succ P_5 \succ P_6 \succ P_8 \succ P_7 \succ P_1 \succ P_4$	0.0132	0.9995
9	$(-2.9901, -3.8106, -3.7685, -2.7448, -3.3224, -3.2793, -3.0657, -3.1419)$	$P_2 \succ P_3 \succ P_5 \succ P_6 \succ P_8 \succ P_7 \succ P_1 \succ P_4$	0.0129	0.9995
10	$(-2.9891, -3.8112, -3.7689, -2.7433, -3.3208, -3.2810, -3.0651, -3.1407)$	$P_2 \succ P_3 \succ P_5 \succ P_6 \succ P_8 \succ P_7 \succ P_1 \succ P_4$	0.0126	0.9995

Table 17 shows that varying q over the entire tested range leaves the ranking completely unchanged, and the correlation with the base case never drops below 0.9995: the non-membership exponent has almost no bearing on the outcome once p is fixed, at least for the present dataset.

Table 18
Sensitivity analysis for p (with $q = 3$ fixed)

p	Scores	Ranking	Std. Dev.	Correlation
2	$(-2.9875, -3.8433, -3.7626, -2.7379, -3.3141, -3.2698, -3.0629, -3.1396)$	$P_2 \succ P_3 \succ P_5 \succ P_6 \succ P_8 \succ P_7 \succ P_1 \succ P_4$	0.0000	1.0000
3	$(-2.5415, -3.2790, -3.2400, -2.4456, -2.8129, -2.8181, -2.6592, -2.8281)$	$P_2 \succ P_3 \succ P_8 \succ P_6 \succ P_5 \succ P_7 \succ P_1 \succ P_4$	0.0906	0.9848
4	$(-2.3208, -2.9288, -2.9360, -2.3189, -2.5306, -2.5587, -2.4462, -2.6869)$	$P_3 \succ P_2 \succ P_8 \succ P_6 \succ P_5 \succ P_7 \succ P_1 \succ P_4$	0.1627	0.9326
5	$(-2.2221, -2.7214, -2.7464, -2.2588, -2.3567, -2.3974, -2.3207, -2.6161)$	$P_3 \succ P_2 \succ P_8 \succ P_6 \succ P_5 \succ P_7 \succ P_4 \succ P_1$	0.2132	0.8500
6	$(-2.1665, -2.6777, -2.6216, -2.2293, -2.2435, -2.2893, -2.2417, -2.5791)$	$P_2 \succ P_3 \succ P_8 \succ P_6 \succ P_5 \succ P_7 \succ P_4 \succ P_1$	0.2353	0.7798
7	$(-2.1357, -2.6522, -2.5337, -2.2142, -2.1641, -2.2401, -2.1911, -2.5597)$	$P_2 \succ P_8 \succ P_3 \succ P_6 \succ P_4 \succ P_7 \succ P_5 \succ P_1$	0.2532	0.7151
8	$(-2.1219, -2.6457, -2.4626, -2.2119, -2.1029, -2.2342, -2.1635, -2.5548)$	$P_2 \succ P_8 \succ P_3 \succ P_6 \succ P_4 \succ P_7 \succ P_1 \succ P_5$	0.2698	0.6510
9	$(-2.1146, -2.6397, -2.4093, -2.2100, -2.0551, -2.2317, -2.1458, -2.5518)$	$P_2 \succ P_8 \succ P_3 \succ P_6 \succ P_4 \succ P_7 \succ P_1 \succ P_5$	0.2836	0.5955

Table 18 tells a different story: the ranking already loosens at $p = 3$ (P_5 and P_8 trade places) and keeps drifting as p grows, with the correlation coefficient falling steadily from 1.0000 to 0.5955 by $p = 9$. The leading pair $\{P_2, P_3\}$ and the trailing platform P_4 remain broadly recognisable throughout, but the middle of the order reshuffles substantially, and by $p \geq 7$ even P_1 and P_5 exchange their relative standing. Figure 3 plots the correlation coefficient against the tested values of p and q side by side and makes the asymmetry visually explicit.

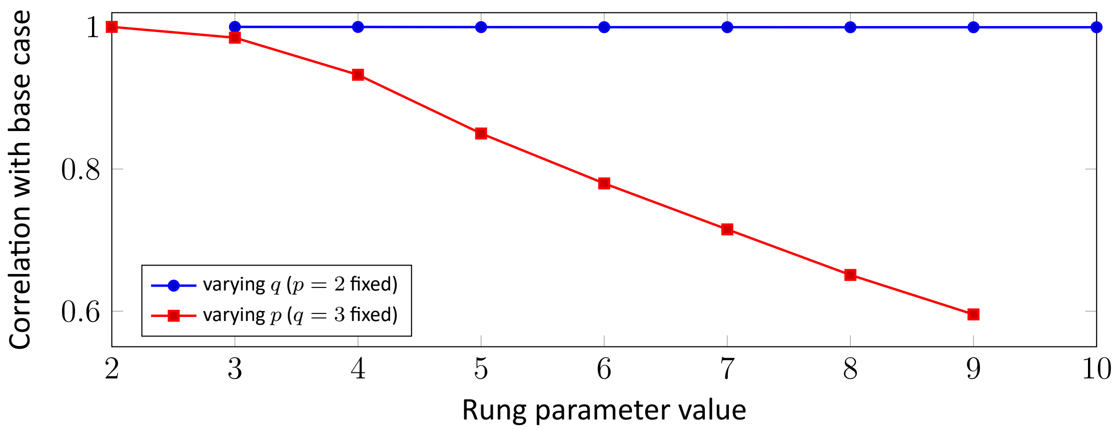

Fig. 3. Correlation with the base-case ranking as p and q are varied independently

Overall, the results indicate that the ranking produced by the proposed LP model is markedly more sensitive to the membership rung p than to the non-membership rung q: an analyst who is unsure which value of q to adopt can do so with little risk of overturning the outcome, whereas the choice of p deserves closer attention, since larger memberships exponents progressively compress the membership grades and thereby reshape the relative standing of the middle-tier platforms. This asymmetry is a natural consequence of allowing membership and non-membership their own rungs, and would remain invisible under the single-rung IVq-ROF formulation.

7. Concluding Remarks

IVpqROFS, carrying two independent rung parameters, provide a flexible medium for representing membership and non-membership information under uncertainty. This study generalized the notions of mean, variance, covariance, and correlation coefficient to the IVpq-ROF setting, with accompanying theorems establishing their symmetry, boundedness, and behaviour under scalar transformation. Building on these measures, DM weights were derived directly from each expert's correlation with group-level outlier profiles, and a DEA-inspired linear programme was formulated to rank alternatives under a common acceptability threshold. The framework was applied to a rural telehealth-platform selection problem, identifying P_2 as the leading platform, and the resulting ranking was validated through an integrated CRITIC and TODIM procedure alongside a Copeland fusion of the per-DM rankings, both of which reproduced the leading and trailing platforms while reshuffling the middle tier. A sensitivity analysis further showed the ranking to be largely insensitive to the non-membership rung q but noticeably more responsive to the membership rung p, a distinction a single-rung model cannot reveal.

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Conflicts of Interest

The authors declare that they have no conflict of interest.

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